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A MODEL OF INTERTEMPORAL ASSET PRICES UNDER ASYMMETRIC INFORMATION

Jiang Wang

Alfred P. Sloan School of Management
Massachusetts Institute of Technology
50 Memorial Drive
Cambridge, MA 02139

[no 3261]

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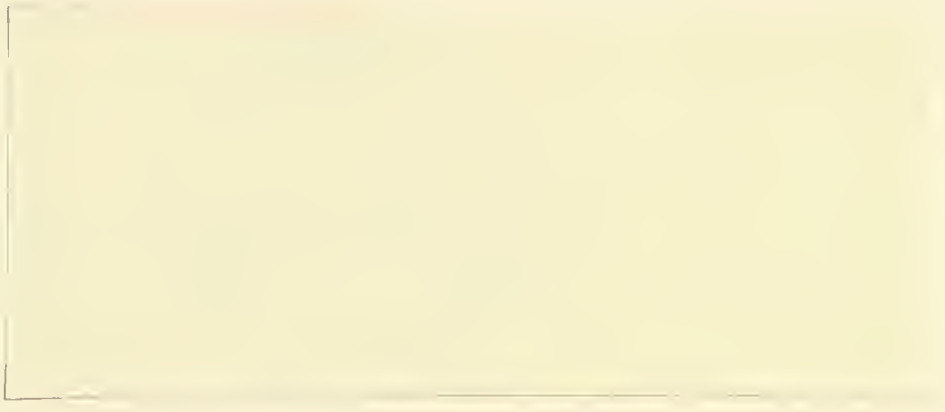

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ABSTRACT

This paper presents a dynamic asset pricing model under asymmetric information. Investors have different information concerning the future growth rate of dividends. They rationally extract information from prices as well as dividends and maximize their expected utility. The model has a closed-form solution to the rational expectations equilibrium. We find that existence of uninformed investors increases the risk premium. Supply shocks can affect the risk premium only under asymmetric information. Information asymmetry among investors can increase price volatility and negative autocorrelation in returns. Less informed investors may rationally behave like price chasers.

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I. INTRODUCTION

In this paper, we present a general equilibrium model of dynamic asset pricing under asymmetric information. We consider an economy endowed with a given quantity of equity. The dividends on the equity grow at a stochastic rate. The investors in the economy can be either informed or uninformed: the informed investors know the future dividend growth rate, while the uninformed investors do not. All the investors observe current dividend payments and stock prices. Since the growth rate of dividends determines the rate of appreciation of stock prices, changes of prices provide signals about the future growth of dividends. The uninformed investors rationally extract information about the state of the economy from the prices as well as dividends. Because we assume an incomplete market structure, the signals do not fully reveal the true values of all the state variables. In equilibrium, investors with access to different information will anticipate different expected returns from investing in the stocks.

We explore the implications of our model for the behavior of stock prices, risk premia, price volatility, autocorrelation in stock returns and investors' trading strategies.

Under asymmetric information, the uninformed investors learn about the future dividend growth rate from dividends and prices. Their estimate of the dividend growth rate will affect their demand of the stocks, hence the equilibrium prices. The errors of the uninformed investors in estimating the dividend growth rate causes prices to deviate from their fundamental values. However, the flow of dividends provides a flow of new information about the underlying growth rate. The uninformed investors will continuously update their estimate based on the newly arrived information and correct the errors

made in previous estimation. Therefore, the estimation errors of the uninformed investors are only temporary.

One application of our model revolves around the problem of equity premia.[1] For example, Mehra and Prescott (1985) argue that existing models of asset pricing with identical investors, perfect information and time-additive preferences are inadequate in explaining the high equity premia observed in the U.S. stock market.[2] Abel (1988) suggests that heterogeneity in investors' beliefs may help us to understand the high risk premia. Presumably, investors' bebiefs are generated by the information they have. Cur model provides a rational framework to study the impact of differential information on risk premia. We show that the existence of uninformed investors can lead to risk premia much higher than that under symmetric and perfect information. When more investors are less informed in the economy, prices will contain less information about the fundamentals of the stock. Hence, there will be more uncertainty concerning the future payoffs from the stocks. Higher premia will be demanded by the less informed investors to invest in the stocks. As the fraction of the uninformed investors in the economy increases the risk premia on stocks also increases.

De Long et. al. (1990) have suggested that "noise trading" in the market (or "supply shocks" in the terminology of this paper) could increase the price volatility, hence the risk of investing in the stock market and the risk premia. The basic assumption that drives their result is that some investors have finite investment horizons. The current model assumes infinite horizon for all investors. It is, then, shown that the unconditional expected excess

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[1] See Grossman and Shiller (1981), Mehra and Prescott (1985).

[2] More general preferences have been suggested in explaining the observed premium. See, for example, Epstein and Zin (1989a,b), Constantinides (1990), Heaton (1990).

return from the stocks only depends on the risk concerning the stocks' future cash flows, which we will call the fundamental risk of the stocks. Even though noise trading does move prices and incease the price volatility, without information asymmetry it does not change the fundamental risk of the stocks. However, when information asymmetry is present in the economy, noise trading will affect the information quality of prices in revealing private information about future cash flows. More noise trading makes price less informative about the future dividend growth. This may increase the uncertainty of future cash flows to the less informed investors, hence the required the return. Therefore, in our model only under information asymmetry, noise trading can affect the risk premia.

Many authors have argued that a typical asset pricing model with identical investors and perfect information cannot reconcile the large volatility of stock prices observed in the market with the history of smooth dividends.[3] Campbell and Kyle (1988) suggest that the existence of noise trading in the market can help in explaining the high volatility in stock prices. In our model, we show that the imperfect information of some investors will cause stock prices to be more volatile, compare with the case when all investors have perfect information. Unable to observe the true growth rate of dividends, the uninformed investors use dividends and prices to make inferences. They will rationally attribute a change in dividend to reflect partially a change in dividend growth rate even when there is no change in it. More precisely, the innovations to the uninformed investors' estimate of the dividend growth rate will be positively correlated with dividend shocks. This of course will feed back to the prices through their demand. Hence, small changes in

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[3] See LeRoy and Porter (1981) and Shiller (1981). See also Flavin (1983), Kleidon (1986), Marsh and Merton (1986), Mankiw, Romer and Shapiro (1985). For more recent work, see Campbell and Shiller (1988), West (1988).

dividends can cause large changes in prices. However, we have also found that the innovation variance of prices does not always monotonocally increase with the fraction of uninformed investors. In other words, adding more informed investors to the economy can sometimes destablize the prices and increase the price volatility. This is due to the effect of asymmetric information. As informed investors are introduced into the economy, more information about the fundamentals will be available to the market through the demand of the informed investors and the price. This should have the effect of reducing the price volatility. At the same time, information asymmetry introduces heterogeity among the investors. Less information will be revealed by the prices about non-fundamentals, such as the level of supply shocks. This can cause price to actually become more volatile.

Recent empirical studies suggest that there may exist significant negative serial correlation in long horizon stock returns.[4] Assuming certain stationarity of the economy, we can have stock returns negatively serially correlated due to mean reversion in some underlying state variables that affects the expected excess returns. However, when information asymmetry is present, the less informed investors can only learn about these state variables from realized returns. This will increase the dependence of their expected future return on past returns and generate additional negative serial correlation in stock returns.

We also apply our model to analyze the trading strategies that are used by investors with different information sets. When information asymmetry is present, the informed investors hedge against not only possible changes in the underlying state variables but also the reaction of the uninformed investors.

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[4] See Fama and French (1987), Poterba and Summers (1988), Lo and MacKinlay (1988). Also see Richardson (1988).

In other words, they take advantage of the errors made by the less informed investors to make profits. The uninformed investors trade based on their information extracted from prices and dividends. We find that in some cases, the uninformed investors rationally adopt trading strategies that look like price chasing: they buy stocks when prices go up and sell when prices go down. A number of studies present evidence consistent with this type of trend-chasing strategy.[5] Our results show that trend-chasing like behavior can be rational for the less informed investors under asymmetric information while the informed investors behave as contrarians.

There are two major obstacles in tackling the problem of dynamic asset pricing under asymmetric information. One is the notion of "no trading" and/or "fully revealing",[6] and the other is the mathematical difficulty involved in solving the system. The "no-trading theorem" states that if asymmetric information is the only motivation for trading, then by his willingness to trade an investor reveals his information to the market. Hence, information asymmetry will be eliminated in equilibrium and no trade will actually occur as new information comes in. This result crucially depends on the market structure of the economy (see, e.g., Grossman (1977, 1981)). Under incomplete markets, there can be motivations other than the arrival of new information that cause investors to trade and the argument for the irrelevancy of diverse information breaks down. When information asymmetry is coupled with an incomplete market structure, the problem becomes mathematically very involved.[7] The current model, to our knowledge, is the first dynamic asset

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[5] See, Anderassen and Kraus (1988), Case and Shiller (1988), Frenkel and Froot (1988) and Shiller (1987). See also Soros (1987).

[6] See Grossman (1981), Milgrom and Stoky (1985).

[7] See Duffie and Huang (1986) and He and Pearson (1988).

pricing model under asymmetric information which provides a closed-form solution.[8]

The formal model is spelled out in section II. In section III, we consider the traditional situation in which investors have homogeneous and perfect information about the economy. This provides a benchmark case for our economy. A rational expectations equilibrium of the full model is obtained in section IV by sequentially solving the problems of investors' rational learning, optimization and market equilibrium. In section V, we discuss the equilibrium price function and how, under asymmetric information, prices deviate from their perfect information benchmark value. The problems of risk premia and price volatility are addressed in section VI and VII respectively. In section VIII, we study the autocorrelation in stock returns. In section IX, we analyze the optimal investment strategies followed by investors with different information. Some further comments are provided in section X. Section XI concludes.

II. THE ECONOMY

We consider a simple economy with a single physical good. The economy is defined as follows.

ASSUMPTION A1 The economy is endowed with certain amount of risky capital.

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[8] The information structure in this paper is similar to the one considered by Townsend (1981). Singleton (1985), Carino (1986) considered models in which all private information becomes public after a short period (one or two period) so that effectively the less informed investors' learning problem becomes static. In this paper, we allow private information to remain private. Therefore, the less informed investors' learning problem becomes dynamic which will generate interesting results conserning optimal investment strategies and equilibrium prices.

Each unit of the risky capital generates a flow of output (dividend) at an instantaneous rate D. D is governed by the diffusion process:[9]

$$dD = a_D(\mu - kD)dt + \sigma_D dw_D, \tag{2.1}$$

where μ is a state variable following an Ornstein-Uhlenbeck (O-U) process

$$d\mu = a_\mu(b - \mu)dt + \sigma_\mu dw_\mu. \tag{2.2}$$

Here, w_D, w_μ are two independent standard Wiener processes, and a_D, a_μ, b, k, σ_D, σ_μ are positive constants.

Clearly, $a_D(\mu - kD)$ gives the expected growth rate of dividends. When $k>0$, μ/k can be interpreted as the short-run steady state level for the dividend rate D, and it fluctuates around a long-run steady state level b/k. (D,μ) is a Gaussian Markov process.

ASSUMPTION A2 Let the total amount of risky capital be $1+\theta$. θ satisfies the stochastic differential equation:[10]

$$d\theta = -a_\theta \theta dt + \sigma_\theta dw_\theta, \tag{2.3}$$

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[9] With this form of the dividend process, D can take negative values. However, one can make the probability of D reaching negative values be as small as one likes by choosing the parameter values. One interpretation for negative dividends is that investors have to put back some investments to maintain the future cash flow.

[10] An equivalent interpretation to the stochastic supply is the existence of noise traders with their demand of the stock being $-\theta$.

where a_θ, σ_θ are positive constants and w_θ is a standard Wiener process. For simplicity, we assume that w_θ is uncorrelated with w_D and w_μ.[11]

By A2, the total supply of the risky capital is assumed to be stochastic. It has a long-run stationary level which is normalized to 1. θ gives the deviation of the current supply from its long-run stationary level.

ASSUMPTION A3 Each unit of the risky capital is represented by one perfectly divisible share of the stock held by the investors in the economy. Shares are traded in a competitive stock market with no transactions costs. The stock is the only security that is traded in the market. Let S be the equilibrium price of the stock.[12]

ASSUMPTION A4 There is a risk-free storage technology available to the economy with a constant rate of return 1+r (r>0). All investors have access to the storage technology at no cost.[13]

In order to have incomplete markets, we have introduced an additional state variable by assuming the aggregate quantity of the risky capital to be stochastic. This is only for the simplicity in exposition. The incompleteness of the market can be modeled in a rational, self-contained framework without

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[11] The independence among all the shocks to the economy is assumed purely for algebraic simplicity. One can easily extend the model to the case of more general correlation structures.

[12] We will not allow free disposal of the stocks. Therefore, the shares in this model do not have limited liability. This would allow the price of the stock to take negative values.

[13] A general equilibrium justification for a constant interest rate in an economy with CARA preferences is given by Sundaresan (1983).

making ad hoc assumptions about the stock supply or investors' behavior. In particular, Wang (1989, 1990) has generated the same result by assuming rational investors, fixed stock supply and possibilities of risky production. We have also restricted the shares of equity to be the only security traded in the market.[14]

ASSUMPTION A5 There are two types of investors in the economy: the informed and the uninformed. The informed investors have perfect private information about the state variables μ and θ, while the uninformed investors have no private information about them. All investors observe the price of the stock and dividends. Hence, $I_t=\{D_\tau,\mu_\tau,\theta_\tau,S_\tau:\tau\leq t\}$ is the informed investors' information set at time t and $I_t^*=\{D_\tau,S_\tau:\tau\leq t\}$ is the uninformed investors' information set. Let the fraction of the uninformed investors be ω.

ASSUMPTION A6 The structure of the economy is common knowledge to all the investors in the economy.

By A5, we have assumed that investors' information about the state variables can be imperfect.[15] In addition, we allow investors to differ in their information about the economy. Given the structure of the markets and the economy, S and D will not be sufficient to reveal the values of both μ and θ to the uninformed investors. Information asymmetry will persist in the equilibrium.

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[14] This assumption makes the capital market dynamically incomplete in the sense of Harrison and Kreps (1979).

[15] For earlier work on the optimal portfolio choice and asset prices under imperfect information, see, e.g., Merton (1971), Dothan and Feldman (1986), Detemple (1986), Gennotte (1986).

ASSUMPTION A7 Investors choose consumption and investment policies in order to maximize expected utilities conditioned on their information sets, $E[\int u(c(\tau),\tau)d\tau|\cdot]$. All investors have preferences exhibiting constant absolute risk aversion (CARA):[16]

$$u(c(t),t) = -\exp(-\rho t - c(t)), \tag{2.4}$$

where ρ is the time impatience parameter and $c(t)$ is the consumption rate at time t.

The CARA preferences are assumed so that a closed-form solution to the model can be obtained. With CARA preferences, an investor's asset demand will be independent of his wealth.[17] This implies that the equilibrium price of the stock well be independent of the wealth distribution of the investors as well as the level of aggregate wealth. This independence greatly simplifies our problem.

III. THE BENCHMARK CASE: UNDER PERFECT INFORMATION

Before we solve the full model specified in the previous section, let us first consider the special case in which all investors are informed (i.e.

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[16] Here, we have assumed all investors to have the same risk aversion and the Arrow-Pratt measure to be 1. This is because we want to purely focus on the effect of information on stock prices. This assumption can be easily relaxed as long as we remain in the CARA class. See the discussion later.

[17] With CARA utility, negative consumption and negative wealth are possible. In this paper, we do not impose non-negativity constraint to rule out negative wealth.

ω=0).[18] This case corresponds to the traditional scenario which assumes identical investors and perfect information.[19] The equilibrium price under perfect information provides a measure about the fundamental value of the stock. The corresponding price volatility and risk premium also tell us what they should be in the framework of a representative investor with perfect information.

Under perfect information, the price of the stock depends only on the primary state variables D, μ and θ. Given the CARA preferences and the linear processes governing D, μ and θ, the price is a linear function of the state variables. Let the equilibrium stock price under perfect information be S_0. One can show that[20]

$$S_0 = \Phi + (s_{00}+s_{\theta 0}\theta), \tag{3.1.1}$$

where

$$\Phi = E_t \int_t^{\infty} e^{-rt} D(s)ds = (a_\mu b s_{\mu 0}/r)+s_{D0}D+s_{\mu 0}\mu. \tag{3.1.2}$$

Here $s_{D0}=1/(r+a_D k)$, $s_{\mu 0}=a_D s_{D0}/(r+a_\mu)$, $s_{00}=-(s_{D0}^2\sigma_D^2+s_{\mu 0}^2\sigma_\mu^2)<0$ and $s_{\theta 0}$ is also negative.

Φ gives the present value of expected future cash flows discounted at the risk-free rate. Risk aversion of the investors increases the expected return to the stock by subtracting a term from the price rather than by increasing

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[18] This is similar to the case considered by Campbell and Kyle (1988).

[19] See, for example, Lucas (1978), Cox, Ingersoll and Ross (1985).

[20] This is a special case of the general model. The proof of this result can be obtained from the solution to the general case, which is provided in the following section.

the discount rate. Indeed, $s_{00}+s_{\theta 0}\theta$ ($s_{00}, s_{\theta 0}<0$) represents the discount on the price of the stock to compensate for the risk in its future cash flow. It increases with θ, the supply shock of the risky capital, because θ determines the total amount of aggregate risk the economy is exposed to. As θ increases, each investor has to bear more of the market risk in equilibrium by holding more stocks. The price of the stock has to adjust to give higher expected return in order to induce investors to invest more in the stock.

Given the equilibrium price of the stock, the conditional expected excess return to a share of stock at time t is

$$E_t[dS+Ddt-rSdt]/dt = [-rs_{00}-(r+a_\theta)\theta]. \tag{3.2}$$

The unconditional expected excess return is $e_{00}\equiv -rs_{00}$. e_{00} depends only on σ_D and σ_μ which characterize the "fundamental risk" associated with future cash flows of the stock. Note that e_{00} is independent of σ_θ. Therefore, the size of the supply shocks does not affect the unconditional expected excess return, even though it does affect the price volatility. This result contrasts with the result of De Long et. al. (1990). It is assumed in De Long et. al. that rational investors have finite horizons and have to liquidate their position at the end of their lifetime. Since supply shocks will cause temporary shifts in the price from its "fundamental value", investors with finite lifetime will face additional risk in price due to supply shocks when they have to liquidate their stock holding. Hence, they will demand extra premium for the addtional risk in liquidation prices due to supply shocks. For investors with longer horizons, the effect of the additional risk of liquidation prices due to supply shocks will be smaller. When investors have infinite horizons, the unconditional expected excess return is determined only by the fundamental

risk of the stock which is the risk in its future cash flows. More volatile prices caused by temporary shocks in supply does not change the risk of investing in the stock and the risk premium. As we will see later, this in no longer the case under asymmetric information.

The innovation variance of the stock price (i.e., the variance of instantaneous stock return) is a constant given by

$$\sigma_{SO}^2 = s_{DO}^2\sigma_D^2+s_{\mu O}^2\sigma_\mu^2+s_{\theta O}^2\sigma_\theta^2. \tag{3.3}$$

A constant variance of prices implies that the variance of the percentage returns increases as the price of the stock decreases and vice versa (see Campbell and Kyle (1988)). This phenomenon has been noted in the U.S. stock market data (see Black (1976), Nelson (1987)).

When both the informed and the uninformed investors are present in the economy, the situation becomes more complicated. However, we may expect that the equilibrium price will somewhat resemble the functional form of the above benchmark case due to the underlying linearity in the model.

IV. EQUILIBRIUM OF THE ECONOMY

In this section, we solve for the equilibrium of the economy described in section II. The equilibrium concept is that of the rational expectations developed by Lucas (1972), Green (1973), Grossman (1976), and Kreps (1977). In such an equilibrium, investors maximize their expected utilities based on their information, the markets clear, and investors' expectations about the distributions of all the random variables are self-fulfilling.

The equilibrium stock price depends on all the state variables of the economy. The primary state variables are the current dividend rate D, the

short-term stationary level of the dividend μ and the aggregate supply shock of the stock θ. However, when information asymmetry is present in equilibrium these primary state variables are not sufficient to fully describe the state of the economy. The uninformed investor do not directly observe μ and θ. They can only use dividends and prices to infer the true values of μ and θ. Their demand of the stock will depend on the inferred values (estimates) of μ and θ. Therefore, the existence of the uninformed investors introduces new state variables into the economy, such as the estimators they use. We call these new state variables "induced state variables".

The way we obtain an equilibrium of the economy is similar in essence to that of Grossman (1976) and others in a static setting.[21] We first conjecture an equilibrium price function. Based on the assumed price function, we solve the investors' learning and optimization problem. Market clearing is then imposed to verify the conjectured price function.[22]

We can write $S=S(D,\mu,\theta,\cdot)$ where "$\cdot$" denotes the induced state variables. The observations of the perfect information case in the previous section suggest the following proposition.

PROPOSITION For the economy defined by A1-A7, there exists a rational expectations equilibrium. The equilibrium price function has the following linear form:

$$S = \Phi + (s_0+s_\theta\theta) + s_\eta(\hat{\mu}-\mu), \tag{4.1}$$

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[21] See, for example, Hellwig (1980), Diamond and Verrecchia (1981), Amati (1985).

[22] The uniqueness of equilibrium will not be discussed in this paper.

where $\hat{\mu}=E[\mu|I_t^*]$ is the Kalman filter for μ used by the uninformed investors based on their information set I_t^*.

The rest of this section will provide a proof of the PROPOSITION following the approach specified earlier.

Because an uninformed investor's demand of the stock depends on his estimates of the unobserved state variables, $\hat{\mu}$ is in the price function as an induced state variable. By the same token, one might expect $\hat{\theta}$, the filter for θ, to be also in the price function. However, this is unnecessary. Define $s_\mu=s_{\mu 0}-s_\eta$ and $\xi=s_\mu\mu+s_\theta\theta$. We have the following lemma:

LEMMA Given the observations of D and S, the filters of an uninformed investor for μ and θ, $\hat{\mu}$ and $\hat{\theta}$ respectively, satisfy the condition:

$$s_\mu\hat{\mu}+s_\theta\hat{\theta} = s_\mu\mu+s_\theta\theta, \text{ or } s_\mu(\mu-\hat{\mu}) = -s_\theta(\theta-\hat{\theta}). \tag{4.3}$$

Proof: $E[\xi|I_t^*]=s_\mu\hat{\mu}+s_\theta\hat{\theta}$. Observing D, S and knowing $\hat{\mu}$, an uninformed investor can deduce ξ. Hence, ξ is in the information set I_t^* and $E[\xi|I_t^*]=\xi$.

Q.E.D.

Therefore, $\hat{\theta}$ is functionally related to the state variables already included in Eq.(4.1). The estimation errors of the two variables are linearly related. For ξ, we have:

$$d\xi = [a_\mu s_\mu(b-\mu)-a_\theta s_\theta\theta]dt + \sigma_\xi dw_\xi, \tag{4.4}$$

where $\sigma_\xi^2 \equiv s_\mu^2 \sigma_\mu^2 + s_\theta^2 \sigma_\phi^2$ and dw_ξ is a standard Wiener process defined in Appendix A.

1. The filtering problem of the uninformed investors

An uninformed investor learns about the values of μ and θ from his observations of D and S. His optimal estimators for μ and θ based on his information set $I_t^* = \{D_\tau, S_\tau : \tau \le t\}$ are derived in Appendix A. The results are summarized in the theorem below:

THEOREM 4.1 Given $I_t^* = \{D_\tau, S_\tau : \tau \le t\}$, the filters $\hat{\mu}$ and $\hat{\theta}$ satisfy the following stochastic differential equations:

$$\begin{bmatrix} d\hat{\mu} \\ d\hat{\theta} \end{bmatrix} = \begin{bmatrix} a_\mu(b-\hat{\mu}) \\ -a_\theta \hat{\theta} \end{bmatrix} dt + \begin{bmatrix} h_{\mu D} & h_{\mu\xi} \\ h_{\theta D} & h_{\theta\xi} \end{bmatrix} d\bar{w} , \tag{4.5}$$

$$d\bar{w} = \begin{bmatrix} dD - a_D(\hat{\mu} - kD)dt \\ d\xi - a_\mu s_\mu (b-\hat{\mu})dt + a_\theta s_\theta \hat{\theta} dt \end{bmatrix}, \tag{4.6}$$

where $h_{\mu D}$ (>0), $h_{\mu\xi}$, $h_{\theta D}$ (>0), $h_{\theta\xi}$ are constants given in Appendix A. $\bar{w}$, the innovation process of the filters, is a standard Wiener process with respect to $F^{D,S}(t)$, the σ-algebra generated by I_t^*. Furthermore, the information strucuture generated by $F^{\bar{w}}(t)$ is equivalent to the one generated by $F^{D,S}(t)$.

As specified in Eq.(2.1), μ determines the growth rate of dividends. Not observing μ, the uninformed investors rationally draw inference about μ from dividends and prices. They will rationally attribute a shock in dividend to reflect partially a shift in the growth rate of dividends. This gives rise to the positive contribution of dividend shocks to innovations of $\hat{\mu}$, as shown in Eq.(4.5). Thus, $E[d\hat{\mu} dD | I_t^*] = h_{\mu D} > 0$ even though dw_D and dw_μ are independent.

From Eq.(4.6), we observe that $\hat{\theta}$ also positively responds to dividend shocks. For the uninformed investors, a positive innovation in D suggests an increase in μ. Conditioned on $\xi=s_{\mu}\mu+s_{\theta}\theta$, the increase in μ must be offset by an increase in θ (since s_{θ} is negative). Hence, $E[d\hat{\theta}dD|I_t^*]=h_{\theta D} > 0$ although θ and D are independent. The joint estimation of μ and θ based on both D and S generates the induced correlation between the two filters and dividends and prices. The induced correlation between the uninformed investors' estimates and dividends and prices are very important in understanding the behavior of stock prices and returns.

For future convenience, define $\eta=\hat{\mu}-\mu$ to be the uninformed investors' error in estimating μ. As derived in Appendix A,

$$d\eta = -a_{\eta}\eta dt + (h_{\mu D}\sigma_D dw_D + h_{\mu\xi}\sigma_{\xi} dw_{\xi} - \sigma_{\mu} dw_{\mu}), \tag{4.7}$$

where a_{η} is a positve constant. Hence, η follows a standard O-U process. The fact that η is mean-reverting to zero implies that the estimation error of the uninformed investors are only temporary. As a matter of fact, the continuous flow of dividends as well as changes in prices provides a flow of new information about the underlying growth rate of dividends. The uninformed investors constantly update their estimates about μ (and θ) based on the newly arrived information and correct the errors made in their previous estimation. Suppose that there has been a positive shock in D while there has been no shock to μ. Not observing μ, the uninformed investors will rationally attribute the dividend shock to partially reflect a high μ, hence increase their estimate $\hat{\mu}$. However, the future dividend will not grow as expected since μ has not increased. When new levels of D are realized, the uninformed

investors will revise their estimate and lower $\hat{\mu}$, eliminating the error in previous estimation.

2. Investment opportunities

Given the process of filter $\hat{\mu}$, the stock price follows the process:

$$dS = [s_{D0}a_D(\mu-kD)+s_{\mu 0}a_\mu(b-\mu)-s_\theta a_\theta\theta-a_\eta s_\eta\eta]dt + \sigma_S dw_S, \tag{4.7}$$

where $\sigma_S^2\equiv(s_{D0}+h_{\mu D}s_\eta)^2\sigma_D^2+(1+h_{\mu\xi}s_\eta)^2\sigma_\xi^2$, and dw_S is a standard Wiener process defined in Appendix A. In order to characterize the investment opportunities in the economy, we consider a zero-wealth portfolio long one share of stock fully financed by borrowing at the risk-free rate. The instantaneous return to this zero-wealth portfolio gives the excess return to one share of stock.

THEOREM 4.2 Given the price process in Eq.(4.7), the undiscounted cumulative cash flow from the zero-wealth portfolio, denoted by Q, satisfies the following stochastic differential equation

$$dQ = (D-rS)dt+dS = [e_0-(r+a_\theta)s_\theta\theta-(r+a_\eta)s_\eta\eta]dt+\sigma_S dw_S. \tag{4.8}$$

where $e_0=-rs_0$.

Hence, the expected excess return on the stock only depends on θ and η. The level of aggregate stock supply affects the excess return on the stock because it determines the total risk exposure of the economy. The short-run stationary level of dividend μ does not affect the excess return per se, but η, the uninformed investors' error in estimating μ, can change the expected excess

return. Theorem 4.2 simplifies the optimization problems of the investors which we now turn into.

3. The optimization problem of the informed investors

For an informed investor, his optimization problem is expressed as follows:

$$0 = \max_{c,x} \{ -\exp(-\rho t-c)+E[dJ(W;D,\mu,\theta,\hat{\mu};t)|I_t]/dt \} \tag{4.9}$$

$$\text{s.t.} \quad dW = [rW+x(D-rS)-c]dt+xdS,$$

$$\lim_{t\to\infty} E[J(W;D,\mu,\theta,\hat{\mu};t)] = 0,$$

where W is his wealth, x his holding of the stock, c his consumption and $J(W;D,\mu,\theta,\hat{\mu};t)$ his value function. We present the solution in the following theorem:[23]

THEOREM 4.3 The program given by Eq.(4.9) has a solution of the form:

$$J(W;D,\mu,\theta,\hat{\mu};t) = -\exp(-\rho t-rW-V(\theta,\eta)), \tag{4.10.1}$$

where $V(\theta,\eta)$ is a quadratic function:

$$V(\theta,\eta) = \frac{1}{2}v_{\theta\theta}\theta^2+\frac{1}{2}v_{\eta\eta}\eta^2+v_{\theta\eta}\theta\eta+v_{\theta}\theta+v_{\eta}\eta+v_0. \tag{4.10.2}$$

The optimal demand of the stock is a linear function of θ and η:

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[23] This problem is similar to the one solved by Merton (1971). The difference is that here we have two state variables governing the excess returns on the stock while Merton only considers the single variable case.

$$x = f_0 + f_\theta \theta + f_\eta \eta, \tag{4.11}$$

where $f_0=[e_0-\sigma_{S\theta}v_\theta-\sigma_{S\eta}v_\eta]/(r\sigma_S^2)$, $f_\theta=[-(r+a_\theta)s_\theta-\sigma_{S\theta}v_{\theta\theta}-\sigma_{S\eta}v_{\theta\eta}]/(r\sigma_S^2)$, and $f_\eta=[-(r+a_\mu)s_\eta-\sigma_{S\theta}v_{\theta\eta}-\sigma_{S\eta}v_{\eta\eta}]/(r\sigma_S^2)$.[24]

Proof: See Appendix B.

As we pointed out earlier, the state of the economy is characterized by four state variables, (D,μ,θ,η). However, the value function of an informed investor only depends on θ and η since they are the only state variables that affect his investment opportunities.

4. The optimization problem of the uninformed investors

For the uninformed investor, the optimazation problem can, in general, be more complicated. The uninformed investors' comsumption-investment policy is generally a function of their information set, which is the whole history of dividends and prices. However, given the processes assumed for the state of the economy as well as the price the information structure generated by I_t^* has an equivalent representation which is one generated by the Wiener process $\bar{w}$ that innovates the filters. The filters are sufficient statistic for I_t^*. Using this equivalent representation of the information structure, one can restate the uninformed investors' optimization problem as a standard Markovian one with the filters being the effective state variables and the innovation process generating the dynamics. It then formally looks similar to the

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[24] In this paper, we use the following notation: $\sigma_{zz'}\equiv E[dzdz'|I_t]$, where z and z' are two random variables.

informed investors' problem. Thus, we have a situation in which the Separation Principle applies.[25]

We can therefore reformulate the uninformed investors' optimization problem as follows:

$$0 = \underset{c^*,x^*}{\text{Max}} \{ -\exp(-\rho t-c^*)+E[dJ^*(W^*;D,\hat{\mu},\hat{\theta};t)|I_t^*]/dt \} \tag{4.12}$$

$$\text{s.t.} \quad dW^* = [rW^*+x^*(D-rS)-c^*]dt+x^*dS,$$

$$\lim_{t\to\infty} E[J^*(W^*,D,\hat{\mu},\hat{\theta};t)] = 0,$$

where W^* is his wealth, x^* his holding in the stock, c^* his consumption and $J^*(W^*;D,\hat{\mu},\hat{\theta};t)$ his value function. Since $E[\eta_t|I_t^*]=0$, from Eq.(4.8) the expected excess return of an uninformed investor only depends on $\hat{\theta}$. The solution to program Eq.(4.12) is given in Theorem 4.4.

THEOREM 4.4 The program given by Eq.(4.12) has a solution of the form

$$J^*(W^*;D,\hat{\mu},\hat{\theta};t)=-\exp\{-\rho t-rW^*-V^*(\hat{\theta})\}, \tag{4.13.1}$$

where

$$V^*(\hat{\theta})= \frac{1}{2} v^*_{\theta\theta}\hat{\theta}^2+v^*_{\theta}\hat{\theta}+v^*_0. \tag{4.13.2}$$

The optimal demand of the stock is

$$x^* = f^*_0 + f^*_{\theta}\hat{\theta}, \tag{4.14}$$

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[25] See Fleming and Rishel (1975). See also Feldman (1984), Detemple (1986), Gennotte (1986).

where $f_0^*=[e_0-\sigma_{S\hat{\theta}}v_\theta^*]/(r\sigma_S^2)$ and $f_\theta^*=[-(r+a_\theta)s_\theta-\sigma_{S\hat{\theta}}v_{\theta\theta}^*]/(r\sigma_S^2)$.

Proof: See Appendix C.

5. Market clearing

Under the assumed form of the price function, the demand of stock by individual informed and uninformed investors are given respectively by Eq.(4.11) and (4.14). When market clears, they must sum to $1+\theta$. Thus

$$(1-\omega)[f_0+f_\theta\theta+f_\eta\eta]+\omega[f_0^*+f_\theta^*\hat{\theta}] = 1+\theta. \tag{4.15}$$

Using Eq.(4.3), we have the following equations:

$$(1-\omega)f_0 + \omega f_0^* = 1, \tag{4.16.1}$$

$$(1-\omega)f_\theta + \omega f_\theta^* = 1, \tag{4.16.2}$$

$$(1-\omega)s_\theta f_\eta - \omega s_\mu f_\theta^* = 0. \tag{4.16.3}$$

Solving Eq.(4.16), we can obtain the coefficients s_0, s_θ, s_η in the price function.[26] This completes our proof of the PROPOSITION.

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[26] (4.16) is a set of non-linear algebraic equations. We are not able to obtain the explicit form of the solution. However, numerical solution can easily be obtained for the coefficients. In the two limiting cases, i.e., $\omega=0$ or $\omega=1$, explicit form of the solution can be calculated. Perturbation methodology can be used in the vicinity of $\omega=0$ and explicit solution of recursive form can be derived.

In what follows, we want to analyze the effect of information on stock prices, risk premium, price volatility and serial correlation in returns. In the current model, the parameter ω characterizes the information structure of the economy. Most of the comparative static analysis are concerned with the effect of changing ω. It is important to realize that ω captures two aspects of the information structure of this model. One aspect is the imperfection in some investors' information. As ω increases more investors have imperfect information and the total amount of information in the market about the fundamentals (i.e., the dividend growth) is less in some sense. The other aspect of the information structure is information asymmetry. As ω changes, the extent to which information is asymmetric among investors also changes. For example, when ω=1 all investors are uninformed and there is no information asymmetry while when ω is lightly less than 1 the investors are no longer homogeneous and information is asymmetric. Information asymmetry can actually cause price to be less informative as we will see later. Hence, as we change ω we are changing these two aspects of the information structure at the same time and the effect we obtain are of course the net of the two.

The effect of imperfect information is best illustrated by comparing the two extreme cases: ω=0 and ω=1. The former corresponds the case of perfect information while the later the case of pure imperfect information. Comparing the result for ω in the vicinity of the two extremes and the extreme case themselves may demonstrate the effect of information asymmetry.

V. STOCK PRICES

As stated in the PROPOSITION, the equilibrium price of the stock is

$$S = \Phi + (s_0 + s_\theta \theta) + s_\eta \eta. \tag{5.1}$$

We can show that when the informed investors dominate in the market (i.e. $\omega \to 1$), s_η is always positive.[27] For reasons discussed in section III, s_θ is negative. Examples are shown in Fig. 1.

The existence of the uninformed investors causes deviations in the stock price from its benchmark value S_0 given in section III:

$$S - S_0 = s_\eta \eta + [(s_0 + s_\theta \theta) - (s_{00} + s_{\theta 0} \theta)]. \tag{5.2}$$

The difference between S and S_0 can be decomposed into two parts. The first part, $s_\eta \eta$, is directly caused by the estimation errors of the uninformed investors. With s_η being positive, the equilibrium price responds positively to the estimation errors of the uninformed investors. The uninformed investors' optimism drives the price up and their pessimism drives the price down. Although the informed investors have perfect information about the errors of the uninformed investors, their risk-aversion prevents them from taking large positions to eliminate these deviations. Since η follows an O-U process and reverts to a zero mean, $s_\eta \eta$ represents a temporary component in price deviations. As discussed earlier, this is because the uninformed investors continuously update their estimates based on the new information from the dividend flow and the prices and the errors made in previous estimation are corrected based on the newly arrived information. The second part is given by the difference between $(s_0 + s_\theta \theta)$ and $(s_{00} + s_{\theta 0} \theta)$. We pointed out earlier that $(s_0 + s_\theta \theta)$ gives the discount in price in order to compensate for the risk of investing in the stock. It represents a permanent deviation in

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[27] This result is obtained by using a perturbation methodology. The proof is available from the author upon request.

the stock price from its benchmark value. Indeed, this second component has a non-zero long-run stationary level of (s_0-s_{00}). As we will see later, the permanent component in the price deviation has to do with the fact that imperfect information of some investors affects the risk of investing in the stock for these investors.

VI. RISK PREMIUM

From Eq.(4.8), the unconditional expected excess return from the stock is given by e_0. As discussed in Section III, e_0 is independent of σ_θ under perfect information (ω=0). Similarly, one can show that the same result holds when ω=1, i.e., all investors are uninformed.[28] Hence, even though the supply shocks do cause the price to vary and to be more volatile, they will not affect the unconditional expected excess return without information asymmetry. However, as shown by Fig. 2, when information asymmetry is present σ_θ will affect the unconditional expected excess return. This is because under asymmetric information, less informed investors rely on prices and dividends to learn about the future dividend growth. As σ_θ increases, the price may become less informative about μ. This will increase the uncertainty in future cash flows perceived by the uninformed investors. Consequently, they will demand a higher return.

The risk premium on the stock is given by $\delta \equiv e/S$ where we only consider the situation when $S > 0$. e is the expected excess return on the stock given by

$$e = e_0-(r+a_\theta)s_\theta\theta-(r+a_\eta)s_\eta\eta. \tag{6.1}$$

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[28] Actually, when all investors are uninformed the equilibrium price will fully reveal θ.

Clearly, δ is not constant over time. Its instantaneous level depends on all the state variables. Let us look at the "long-run level" $\bar{\delta}$ defined by $\bar{\delta}=\bar{e}/\bar{S}$, where $\bar{e}=e_0$ and $\bar{S}$ are, respectively, the long-run stationary level of e and S. Then,

$$\bar{\delta} = \frac{rb}{b+rks_0} - r. \tag{6.2}$$

The information structure affects the long-run stationary level of risk premium only through the long-run level of price, $\bar{S}$. Given the values of the underlying state variables, which govern the future cash flows of the stock, the price of the stock decreases as the fraction of uninformed investors increases (see Fig. 1). The same result holds for the long-run level of the price. Therefore, we conclude that the risk premium on the stock increases with the fraction of uninformed investors. As shown in Fig. 3, $\bar{\delta}$ is an increasing function of ω.

This result is quite intuitive. As we discussed earlier, the risk premium only depends on the fundamental risk of the stock perceived by the investors. As the fraction of uninformed investors increases, the price will contain less information about future dividend growth. This increases the perceived uncertainty about future cash flows by the uninformed investors, hence the risk of investing in the stock. In the equilibrium, a higher premium for the stock will be demanded. It should be clear that this result is mainly driven by the imperfect information the market has about the fundamentals. The information asymmetry plays no important role here.

VII. PRICE VOLATILITY

The process of the equilibrium stock price is given in Eq.(4.7). The price has a constant innovation variance:

$$\sigma_S^2 = (s_{D0}+h_{\mu D}s_\eta)^2\sigma_D^2+(1+h_{\mu\xi}s_\eta)^2\sigma_\xi^2. \tag{7.1}$$

In order to see how the imperfect information of the uninformed investors affects the innovation variance of the price, let us consider the limit case in which all investors are uninformed (i.e. ω=1). In this case, μ drops out of the price function since it is not in the information set of any investors, and in equilibrium the price fully reveals θ. The equilirium price has the following form

$$S_0^* \equiv S(\omega=1) = s_{00}^* + s_{D0}D + s_0\hat{\mu} + s_{\theta 0}^*\theta,$$

where $s_{00}^*=s_0(\omega=1)$, $s_{\theta 0}^*=s_\theta(\omega=1)$. Investors can only extract information about μ from dividends. A shock in D has two effects. Firstly, it affects the current dividend payments. Secondly, it signals a possible change in the future growth of dividends which we call its information effect. Because the stock price increases with dividend as well as with its growth rate, the information impact of dividends causes the price to "overreact" to dividend changes. Small changes in dividend can create large changes in price. This enhances the price volatility. Formally, it is easy to show from Eq.(4.5) that innovations in $\hat{\mu}$ is perfectly correlated with innovations in D. Given the form of the price function, the induced correlation between D and $\hat{\mu}$ leads to more volatile prices.

It is interesting to point out that as ω increases, σ_S^2 does not always

increase monotonically. As shown in Fig. 4, σ_S^2 may peak at some ω in the interior of [0,1]. This reflects the impact of information asymmetry.

It has often been argued in the literature that rational investors with superior information will always stablize prices.[29] The basic reason is that better informed investors will take profitable positions whenever price is out of line with the fundamentals. By doing so, they partially reveal their private information to the market and move the price in the direction of fundamentals. More of these informed investors exist, larger impact they will have on the price and less it will deviate from its fundamental value. Hence, increasing the fraction of informed investors should reduce the price volatility.

Given the information structure in this paper, it is not the case that more informed investors will always stablize the prices. To see why, let us look at the vicinity of ω=1. When ω=1, investors are homogeneous and all uninformed. The absence of information asymmetry enables investors to perfectly infer the level of supply shock from the equilibrium price. In other words, the fact that "no body knows anything" enables everybody to know something. Consider now a small fraction of informed investors are introduced into the market (i.e., ω is slightly less than 1). On the one hand, they will bring in more information about the dividend growth through their demand and prices because they know μ. This has the effect of reducing the price volatility since now the uninformed investors have better information about μ and are less reliable on dividends in learning about μ. On the other hand, the information asymmetry

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[29] In a recent paper, De Long et. al. (1990) argue that if there exist investors in the market who follows, by assumption, certain positive feedback strategies, the introduction of rational speculators can actually destablize prices. Our model differ from theirs in that here all investors' investment strategy are directly derived from utility maximization. Information asymmetry is explicitly modeled to generate the result.

will make price less informative about the level of supply shocks. Price will no longer fully reveal the supply shocks in equilibrium. The uninformed investors have to use past as well as present prices and dividends to infer the current level of θ. The Bayesian updating of the uninformed investors enhances the correlation between innovations in $\hat{\theta}$ and price changes. Appreciation in the stock price (more precisely, an increase in ξ) will be rationally interpreted as partially reflecting an decrease in θ. This implies a higher future return, hence induces higher stock demand by the uninformed investors which feeds back into the prices. Hence, the information asymmetry will make price more volatile. The net effect of increasing the fraction of informed investors will be the sum of the two, which is generally ambiguous. For some parameter values, the effect of information asymmetry could dominate. When more informed investors are introduced in the market, the price can deviate more from the fundamentals and become more volatile.

When a_D, a_μ, a_θ and k are strictly positive, the price of the stock has a finite unconditional variance. The unconditional variance of the stock price, Var[S(t)], is calculated in Appendix E. Fig.5.2 shows that Var[S] can behave in the similar fashion as ω changes.[30]

In various price volatility tests of the stock market efficiency (e.g. LeRoy and Porter (1981), Shiller (1981)), the equilibrium stock prices are obtained by assuming a representative investor with perfect information. The effect of diverse and/or imperfect information have not been taken into consideration. Our model shows that the existence of investors with imperfect

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[30] It should be pointed out that Var[S] can behave in various ways as ω changes, depending on the parameter values. For example, it is even possible to have the situation that the unconditional variance is lower under imperfect information (ω=1) than under perfect information (ω=0). This result should not puzzle us because the unconditional variance of price not only depends on the instantaneous variance-covariance of its components but also depends on all the adjustment coefficients such as a_D, a_μ, a_θ and a_η.

information can cause stock prices to deviate from their fundamental values and to become more volatile. Moreover, information asymmetry can also cause price to be more variable. The traditional models of identical investors and perfect information can understate the price volatility.[31]

VIII. AUTOCORRELATION IN STOCK RETURNS

Recent empirical studies (Fama and French (1987), Poterba and Summers (1988), Lo and Mckinlay (1988)) have found significant negative serial correlation in stock returns over long horizons of 3-5 years. Negative autocorrelation in returns suggests the existence of stationary components in stock prices (Summers (1986)). In this section, we employ our model to study the behavior of stock returns. For convenience, we consider the returns to the zero-wealth portfolio defined in Section IV.[32]

Under asymmetric information, the return to the zero-wealth portfolio is derived in THEOREM 4.2:

$$dQ = (D-rS)dt+dS = [e_0-(r+a_\theta)s_\theta\theta-(r+a_\eta)s_\eta\eta]dt+\sigma_S dw_S.$$

Q has the following solution (see, e.g., Arnold (1974)):[33]

$$Q(t) = Q(t_0) + e_0(t-t_0) + \int_{t_0}^{t} [\sigma_S dw_S(s)-\gamma_\theta s_\theta\sigma_\theta dw_\theta(s)-\gamma_\eta s_\eta\sigma_\eta dw_\eta(s)]$$
$$+ \gamma_\theta s_\theta[\theta(t)-\theta(t_0)] + \gamma_\eta s_\eta[\eta(t)-\eta(t_0)], \qquad (8.1)$$

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[31] See, for example, Mankiw and De Long (1989).

[32] Or equivalently, the excess return to a share of the stock.

[33] Q gives the undiscounted cumulative cash flow from the zero wealth portfolio.

where $\gamma_\theta=(1+r/a_\theta)$, $\gamma_\eta=(1+r/a_\eta)$. Define $\lambda(t)=\gamma_\theta s_\theta\theta(t)$, $\chi(t)=\gamma_\eta s_\eta\eta(t)$ and $Z(t)$ by

$$dZ = \epsilon_0 dt + [\sigma_S dw_S - \gamma_\theta s_\theta \sigma_\theta dw_\theta - \gamma_\eta s_\eta \sigma_\eta dw_\eta]. \tag{8.2}$$

Q can be expressed as the sum of the three components Z, λ and χ:

$$Q(t) = Z(t) + \lambda(t) + \chi(t). \tag{8.3}$$

Clearly, Z(t) follows a random walk with a constant drift and both $\lambda(t)$ and $\chi(t)$ follow standard O-U processes.[34]

In order to look at the autocorrelation in stock returns, let us consider the following ratio:

$$\beta(\tau) = \frac{\mathrm{Cov}[Q(t+\tau)-Q(t),\,Q(t)-Q(t-\tau)]}{\mathrm{Var}[Q(t)-Q(t-\tau)]}. \tag{8.4}$$

$\beta(\tau)$ is calculated in Appendix D.

The numerator in (8.4) is the unconditional covariance between the returns from two consecutive holding periods of length τ:

$$\mathrm{Cov}[Q(t+\tau)-Q(t),Q(t)-Q(t-\tau)] = -\gamma_\xi s_\theta^2 \sigma_\theta^2 \left(1-\frac{r}{a_\theta}\right)\left[\frac{1}{2}+\frac{a_\theta}{a_\theta+a_\eta}s_\eta h_{\mu\xi}\right][G(\tau,a_\theta)]^2.$$

The covariance is negative if $r < a_\theta$ and positive if $r > a_\theta$.[35]

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[34] It should be pointed out that Z(t), $\lambda(t)$ and $\eta(t)$ are not independent (see Appendix D).

[35] When $r > a_\theta$, the financing cost of the zero-wealth portfolio dominates the return, which leads to positive serial correlation in the returns.

Consider the case when $r < a_\theta$, i.e., the covariance is negative. The strong mean-reversion in θ will generate negative autocorrelation in stock returns even under symmetric information. Under asymmetric information, however, the negative autocorrelation will be enhanced. In this case, the uninformed investors use past returns to filter the current level of supply shock. A low return in the past period will be attributed partially to an increase in θ, even when it was caused by a drop in the future dividend growth. Therefore, they will expect a future decrease in θ due to its mean-reverting nature hence a high return. This enhanced correlation between the uninformed investors updating of θ and the realized returns amplifies the serial correlation in the returns. As shown in Fig. 6, information asymmetry can significantly increase the negative autocorrelation in returns.

IX. OPTIMAL INVESTMENT STRATEGIES

In this section, we examine the trading strategies of both the informed and the uninformed investors. As we have seen in the previous sections, investors with different information view the assets differently even in equilibrium. Hence, they will adopt different investment policies.

Given the equilibrium price, THEOREM 4.4 gives the optimal stock demand of an informed investor:

$$x = f_0 + f_\theta \theta + f_\eta \eta. \tag{9.1}$$

As shown in section IV, the excess return are determined by two factors: aggregate supply of the risky capital θ and the uninformed investors' estimation error $\eta = \hat{\mu} - \mu$. They give rise to the two hedging components in the informed investors' demand for stock, as expressed by the last two terms in

Eq.(7.1). Knowledge about expected future revisions in the uninformed investors' estimates does not provide the informed investors with arbitrage opportunities since η is stochastic. However, the informed investors do take advantage of their superior information to make profits by hedging against changes in η. Due to risk-aversion, the hedging position of the informed investors will not be large enough to eliminate the price deviations caused by the uninformed investors.

The optimal investment policy of the uninformed investors is given by Eq.(4.11) in THEOREM 4.3. We rewrite it as

$$x^* = f_0^* + f_\theta^* \hat{\theta}. \tag{9.2}$$

The second part of the demand represents an uninformed investor's hedging demand. $\hat{\theta}$ is the only state variable that may affect the excess return on the stock anticipated by an uninformed investor, hence the variable that he wants to hedge against.

Several empirical studies suggest that some investors in the market behave like price chasers: buy when prices rise and sell when prices drop.[36] In what follows, we show that trend-chasing can be the rational behavior of less informed investors under asymmetric information, as a consequence of our assumptions about the dividend growth rate. In order to study how an uninformed investor's demand for stock responds to changes in prices, we consider the correlation between dx^* and dS conditioned on I_t^*, $E[dx^* dS | I_t^*]$. Given the demand schedule in Eq.(9.2),

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[36] See the discussion in De Long, Shleifer, Summers and Waldmann (1990) and the references therein.

$$E[dx^*dS|I_t^*] = f_\theta^* E[dSd\hat{\theta}|I_t^*] = f_\theta^* \sigma_{S\hat{\theta}}.$$

When f_θ^* is positive, the sign of $E[dx^*dS|I_t^*]$ is the same as that of $E[dSd\hat{\theta}|I_t^*]$. Intuitively, f_θ^* should be positive. When the informed investors infer that θ is high, their holding should also increase. Otherwise, it must be the informed investors who are holding more, which cannot be an equilibrium situation for the uninformed investors.

When the uninformed investors dominate the market (i.e., $\omega \to 1$), the equilibrium price is almost a perfect signal for the supply of the stock. Actually, when all the investors are uninformed ($\omega=1$), the price fully reveals θ. As discussed before, when the aggregate supply of the stock increases, each investor has to hold more stock in equilibrium. The price of the stock has to decrease so that the expected excess return of the stock increases and investors are willing to hold more stocks. This negative correlation between $d\theta$ and dS makes the uninformed investors rationally interpret an observed decrease in the price as partially due to an increase in θ. They, therefore, anticipate a higher excess return from the stock and increase their holding of stock. Hence, when the price provides a good signal for the stock supply, $E[dx^*dS|I_t^*]$ is negative and the uninformed investors behave like contrarians.

However, when the uninformed investors do not dominate the market, the price does not provide a good signal for the supply because of heterogeity among the investors. The sign of $E[dSd\hat{\theta}|I_t^*]$ is ambiguous. As shown by Eq.(4.6) in Section IV, $E[dDd\hat{\theta}|I_t^*]$ is positive while $E[d\xi d\hat{\theta}|I_t^*]$ is negative. A positive $E[dDd\hat{\theta}|I_t^*]$ can be understood as follows. Suppose that there is a positive innovation in D and no innovation in ξ. The uninformed investors will rationally infer a positive shock in μ from the dividend change. Also observing no innovation in $\xi = s_\mu \mu + s_\theta \theta$ with s_θ being negative, the uninformed

investors will infer that θ must have increased as to offset the shift in μ. The joint estimation of μ and θ based on observing D and S gives rise to the induced correlation between D and $\hat{\theta}$. Since both dD and $d\xi$ positively contribute to the changes in the price of the stock. the sign of $E[dSd\hat{\theta}|I_t^*]$ will depend on parameter values. As we show in Fig. 7, $E[dSd\hat{\theta}|I_t^*]$ can be positive for certain ranges of the parameters. This gives the case in which the changes in the uninformed investors demand for stock is positively correlated with the changes in the price as seen from Fig. 8 while the informed investors are acting like contrarians.

Hence, under asymmetric information the less informed investors may rationally behave like price chasers. The information impact of the equilibrium price makes the less informed investors positively respond to price changes. This arises only under asymmetric information.[37]

X. FURTHER DISCUSSION AND COMMENTS

In the current model, we have assumed the aggregate stock supply to be stochastic. This is purely for the simplicity in exposition as mentioned earlier. One can easily think of ways of introducing additional state variables instead of assuming stochastic supply. In a separate paper (Wang (1989)), we have developed a model with only rational investors, risky production and a constant stock supply. Most of the results are formally the same as here.

We have made many simplifing assumptions. Some of these assumptions can be relaxed.

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[37] When investors are equally informed, as in the two limit cases ω=0 and ω=1, this situation does not exist.

For example, we can allow investors to have different degree of risk aversion as long as they remain in the CARA class. The aggregation theorems (see, e.g. Rubinstein (1974)) enable us to reduce the economy to an effective two-person setup used in this paper. We should point out that the results in this paper depends on the assumption of risk-aversion of all investors. It is the risk aversion of the informed investors that prevent them from taking very large positions to take advantage of the uninformed investors' estimation errors. If the informed investors were risk neutral, the only equilibrium in the current competitive set-up would be a fully revealing one. In could be interesting to look at how the risk aversion of the two types of investors affect some of the results.

As for the information structure, one can assume that in addition to dividends and prices the uninformed investors receive some other exogenous signals, which also have the same linearity as mentioned earlier. In addition, we may extend the model to include more than two classes of differently informed investors. When the information sets of different classes of investors have a complete ranking in terms of statistical dominance, the extension would be straight-forward. However, if the ranking is not complete, the situation becomes more complicated.

We have modelled the information structure of the economy in a discrete fashion. There are a finite number of signals available to the investors (e.g., D, S, μ, θ). For each signal, an investor either has access to it or does not have any access. This may introduce certain instabilities into the equilibrium of the economy in terms of the information structure. To illustrate this, let us consider the case in which the population of the informed investors is very small. The price will contain certain information that only the informed investors have and hence provides a valuable source of

information to the uninformed investors. This is true no matter how small the informed investors' population is as long as it is not exactly zero. The structure of the economy as well as the values of all the parameters are common knowledge. Even though the informed investors only have a very small impact on the price,[38] the uninformed investors would have no trouble in identifying the exact perturbation on the price caused by the informed investors. The absolute magnitude of the signal has nothing to do with its information content. Hence, the uninformed investors can still extract the same amount of information from the price. However, when the population of the informed investors is exactly zero, the price does not convey any information held by the informed investors. Therefore, the information content may change abruptly when ω reaches 1. This implies that the limiting equilibrium as $\omega \to 1$ can be drastically different from the equilibrium when $\omega=1$.

In the current setup, however, the instability suggested above is not present. The reason is that when $\omega=1$, price remains a valuable source of information to the uninformed investors because it fully reveals the supply of the stock. The information content is continuous in some sense as ω approaches 1. But we have found examples of instabilities in a variant setup in which prices become informationally valueless when ω is 1.[39] The existence of instabilities in information structure will become an important issue if we try to endogenize the information structure.

XI. CONCLUSION

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[38] This is the result of the competitive assumption and may not be true if investors are allowed to behave strategically.

[39] In the setup developed by Wang (1989), in which hedging against changes in private investment opportunities plays the role of supply shocks, one can show the disruptive change in equilibrium as ω approaches 1.

This paper presents a general equilibrium model of asset pricing under asymmetric information. The model involves investors who are differently informed about the state of the economy. Investors rationally extract information from all the available signals including prices and competitively trade in the market to maximize expected utility. We obtain a closed-form solution to the rational expectations equilibrium of the economy.

We further employ the model to investigate the impact of asymmetric information on equilibrium asset prices, the risk premium, price volatility, autocorrelation is returns and optimal trading strategies. We illustrate that the existence of investors with imperfect information increases risk premia on stocks. We show that the existence of uninformed investors creates temporary deviations in prices from their fundamental values. This causes stock prices to be more volatile. We also find that the informed investors can destablize the price and information asymmetry can increase the price volatility and negative autocorrelation in returns. Another implication of our model is that although supply shocks do make price more volatile, only under asymmetric information the size of supply shocks can affect the unconditional expected excess return. We also show that under asymmetric information, investors with different information adopt different investment strategies. For less informed investors, they may rationally act like price chasers or positive feedback traders.

In a separate work, we use a variant form of the current model to study the behavior of trading volume and its relationship with price behavior.[40]

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[40] Examples of some related empirical work are French and Roll (1985), Barclay, Litzenberger and Warner (1988).

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Appendix A: Solution to the Uninformed Investors' Filtering Problem

In this appendix, we solve the filtering problem of the uninformed investors. Suppose that our system of interest is given by the following stochastic differential equations:

$$dy(t) = [A_0+A_1y(t)+A_2z(t)]dt + Gdw(t), \tag{A.1}$$

$$dz(t) = [B_0+B_1y(t)+B_2z(t)]dt + Hdw(t), \tag{A.2}$$

where $y(t)$ is an n-vector of state variables, $z(t)$ is an m-vector of signals, $dw(t)$ is a k-vector Wiener process, A_0, B_0 A_D, B_1, A_2, B_2, G and H are respectively n×1, m×1, n×n, m×n, n×m, m×m, n×k and m×k matrices of constants. Let $Q=GG'$, $R=HH'$ and $C=GH'$.

Conditioned only on the observations of $z(t)$, the posterior mean of $y(t)$ or the filter, denoted by $\hat{y}_t \equiv E[y|I_t^*]$, is given by the stochastic differential equation:[41]

$$d\hat{y}(t) = [A_0+A_1\hat{y}(t)+A_2z(t)]dt+[P(t)B_1'+C]R^{-1}\{dz(t)-[B_0+B_1\hat{y}(t)-B_2z(t)]dt\}, \tag{A.3}$$

where $P(t)$ is a symmetric matrix given by the solution to the following Riccati equation:

$$\dot{P}(t) = A_1P(t) + P(t)A_1' + Q - [P(t)B_1'+C]R^{-1}[B_1P(t)+C']. \tag{A.4}$$

The innovation process, $\bar{w}(t)$, defined by

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[41] For a complete treatment of the filtering problem, see Liptser and Shiryayev (1977).

$$d\bar{w}(t) = dz(t) - [B_0 + B_1\hat{y}(t) - B_2 z(t)]dt$$

is a Wiener process with respect to $F^z(t)$.[42] We can restate the filtering equation in terms of the innovation process as follows:

$$d\hat{y}(t) = [A_0 + A_1\hat{y}(t) + A_2 z(t)]dt + [P(t)B_1' + C]R^{-1}d\bar{w}. \qquad (A.3')$$

In the filtering problem of the uninformed investors, the signals are D and ξ, where $\xi = s_\mu \mu + s_\theta \theta$. ξ follows the process:

$$d\xi = [a_\mu s_\mu (b-\mu) - a_\theta s_\theta \theta]dt + \sigma_\xi dz_\xi, \qquad (A.5)$$

where $\sigma_\xi^2 = s_\mu^2\sigma_\mu^2 + s_\theta^2\sigma_\theta^2$ and $\sigma_\xi dz_\xi \equiv s_\mu\sigma_\mu dw_\mu + s_\theta\sigma_\theta dw_\theta$ is a standard Wiener process. Define the vector of signals by $z' = [D(t), \xi(t)]$, the vector of state variables by $y' = [\mu(t), \theta(t)]$ and $w' = [w_D(t), w_\mu(t), w_\theta(t)]$. The parameter matrices are defined accordingly:

$$A_0 = \begin{bmatrix} a_\mu b \\ a_\theta \end{bmatrix}, \quad A_1 = \begin{bmatrix} -a_\mu & 0 \\ 0 & -a_\theta \end{bmatrix}, \quad A_2 = 0, \quad Q = \begin{bmatrix} \sigma_\mu^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & s_\mu\sigma_\mu^2 \\ 0 & s_\theta\sigma_\theta^2 \end{bmatrix},$$

$$B_0 = \begin{bmatrix} 0 \\ a_\mu b s_\mu + a_\theta s_\theta \end{bmatrix}, \quad B_1 = \begin{bmatrix} a_D & 0 \\ -a_\mu s_\mu & -a_\theta s_\theta \end{bmatrix}, \quad B_2 = 0, \quad R = \begin{bmatrix} \sigma_D^2 & 0 \\ 0 & \sigma_\xi^2 \end{bmatrix}. \qquad (A.6)$$

Substituting Eq.(A.6) into Eq.(A.3) and (A.4), we will obtain the equations for the filters of the uninformed investors.

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[42] $F^z(t)$ is the smallest σ-algebras with respect to which $z(s)$, $s \le t$, are measurable.

conditions.[3] However, it can easily be shown that $\mathbf{P}(t)$ quickly approaches its steady state limit, which is independent of the initial conditions. Since in this paper we are only interested in the steady state solution of the economy, we will look for the steady state solution to the Riccati equation, denote by

$$\mathbf{P}(\infty) = \lim_{t\to\infty} \mathbf{P}(t) \equiv \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}. \tag{A.7}$$

$\mathbf{P}(\infty)$ satisfies the following equations:

$$-2a_{\mu}p_{11}+\sigma_{\mu}^2 = h_{\mu D}^2\sigma_D^2+h_{\mu\xi}^2\sigma_{\xi}^2, \tag{A.8.1}$$

$$-2(a_{\mu}+a_{\theta})p_{12} = h_{\mu D}h_{\theta D}\sigma_D^2+h_{\mu\xi}h_{\theta\xi}\sigma_{\xi}^2, \tag{A.8.2}$$

$$-2a_{\theta}p_{22}+\sigma_{\theta}^2 = h_{\theta D}^2\sigma_D^2+h_{\theta\xi}^2\sigma_{\xi}^2, \tag{A.8.3}$$

where

$$h_{\mu D}=a_D p_{11}/\sigma_D^2,\ h_{\mu\xi}=(s_{\mu}\sigma_{\mu}^2-a_{\mu}s_{\mu}p_{11}-a_{\theta}s_{\theta}p_{12})/\sigma_{\xi}^2,$$

$$h_{\theta D}=a_D p_{12}/\sigma_D^2,\ h_{\theta\xi}=(s_{\theta}\sigma_{\theta}^2-a_{\mu}s_{\mu}p_{12}-a_{\theta}s_{\theta}p_{22})/\sigma_{\xi}^2.$$

From the LEMMA in Section 4, we have

$$s_{\mu}d\hat{\mu} + s_{\theta}d\hat{\theta} = d\xi. \tag{A.9}$$

Hence, from Eq.(A.9-10) we have

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[3] Of course, normality of the prior distribution should always be maintained.

Hence, from Eq.(A.9-10) we have

$$s_\mu h_{\mu D} + s_\theta h_{\theta D} = 0, \quad s_\mu h_{\mu\xi} + s_\theta h_{\theta\xi} = 1. \tag{A.10}$$

This leads to the steady state solution to the Riccati equation:[44]

$$P_{12} = -(s_\mu/s_\theta)P_{11},\ P_{22} = (s_\mu^2/s_\theta^2)P_{11},$$

$$P_{11} = \frac{\sigma_D^2}{a_D^2\sigma_\xi^2+(a_\mu-a_\theta)^2 s_\mu^2\sigma_D^2}\left[-[a_\mu s_\theta^2\sigma_\theta^2+a_\theta s_\mu^2\sigma_\mu^2] + \right.$$

$$\left. + \sqrt{\sigma_\xi^2[(a_\mu^2 s_\theta^2\sigma_\theta^2+a_\theta^2 s_\mu^2\sigma_\mu^2)+a_D^2 s_\theta^2\sigma_\mu^2\sigma_\theta^2/\sigma_D^2]} \right] \tag{A.11}$$

The filters are expressed as

$$\begin{bmatrix} d\hat{\mu} \\ d\hat{\theta} \end{bmatrix} = \begin{bmatrix} a_\mu(b-\hat{\mu}) \\ -a_\theta\hat{\theta} \end{bmatrix} dt + \begin{bmatrix} h_{\mu D} & h_{\mu\xi} \\ h_{\theta D} & h_{\theta\xi} \end{bmatrix} d\bar{w}, \tag{A.12}$$

where

$$d\bar{w} = \begin{bmatrix} dD - a_D(\hat{\mu}-kD)dt \\ d\xi - a_\mu s_\mu(b-\hat{\mu})dt + a_\theta s_\theta\hat{\theta}dt \end{bmatrix}.$$

Let η be the difference between the filter $\hat{\mu}$ and the true value of the state variable μ. η satisfies the following stochastic differential equation:

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[44] I thank Young-Soo Kim for pointing out a simple solution to the Riccati equation.

$$d\eta = -a_\eta \eta dt + [h_{\mu D}\sigma_D dw_D + h_{\mu\xi}\sigma_\xi dw_\xi - \sigma_\mu dw_\mu], \tag{A.14}$$

where $a_\eta = a_\mu + a_D h_{\mu D} + (a_\theta - a_\mu)s_\mu h_{\mu\xi}$.[45] This is a standard O-U process.

For future use, let us calculate the conditional covariance of each filter with each primary state variable:

$$\sigma_{D\hat{\mu}} = h_{\mu D}\sigma_D^2, \quad \sigma_{\mu\hat{\mu}} = h_{\mu\xi}s_\mu\sigma_\mu^2, \quad \sigma_{\theta\hat{\mu}} = h_{\mu\xi}s_\theta\sigma_\theta^2,$$

$$\sigma_{D\hat{\theta}} = h_{\theta D}\sigma_D^2, \quad \sigma_{\mu\hat{\theta}} = h_{\theta\xi}s_\mu\sigma_\mu^2, \quad \sigma_{\theta\hat{\theta}} = h_{\theta\xi}s_\theta\sigma_\theta^2, \quad \sigma_{\hat{\mu}\hat{\theta}} = h_{\mu D}h_{\theta D}\sigma_D^2 + h_{\mu\xi}h_{\theta\xi}\sigma_\xi^2.$$

Define $\sigma_S^2 = (s_{D0} + h_{\mu D}s_\eta)^2\sigma_D^2 + (1 + h_{\mu\xi}s_\eta)^2\sigma_\xi^2$ and

$$dw_S = (s_{D0} + h_{\mu D}s_\eta)(\sigma_D/\sigma_S)dw_D + (1 + h_{\mu\xi}s_\eta)(\sigma_\xi/\sigma_S)dw_\xi. \tag{A.15}$$

dw_S is a standard Wiener process. From the PROPOSITION, the price follows the following process

$$dS = [s_{D0}a_D(\mu - kD) + s_{\mu 0}a_\mu(b - \mu) - a_\theta s_\theta \theta - a_\eta s_\eta(\hat{\mu} - \mu)]dt + \sigma_S dw_S. \tag{A.16}$$

The covariance of the price with each of the state variables is:

$$\sigma_{SD} = (s_{D0} + h_{\mu D}s_\eta)\sigma_D^2, \quad \sigma_{S\mu} = (1 + h_{\mu\xi}s_\eta)s_\mu\sigma_\mu^2, \quad \sigma_{S\theta} = (1 + h_{\mu\xi}s_\eta)s_\theta\sigma_\theta^2,$$

$$\sigma_{S\hat{\mu}} = (s_{D0} + h_{\mu D}s_\eta)h_{\mu D}\sigma_D^2 + (1 + h_{\mu\xi}s_\eta)h_{\mu\xi}\sigma_\xi^2, \quad \sigma_{S\hat{\theta}} = (s_{D0} + h_{\mu D}s_\eta)h_{\theta D}\sigma_D^2 + (1 + h_{\mu\xi}s_\eta)h_{\theta\xi}\sigma_\xi^2.$$

Appendix B: Solution to the Informed Investors' Optimization Problem

[45] It can easily be shown that a_η is positive.

In this appendix, we solve the optimal investment and consumption problems for an informed investor. Let W be his wealth, c his consumption rate, x his stock holding and $J(W;D,\mu,\theta,\hat{\mu};t)$ his value function. The optimization problem is stated in Eq.(4.9) in section IV. Since the expected excess return on the stock only depends on θ and η, we assume the value function to have the following form:[46]

$$J(W;D,\mu,\theta,\hat{\mu};t) = -\exp\{-\rho t - rW - V(\theta,\eta)\}. \tag{B.1}$$

The Bellman equation is then given as follows:

$$\begin{aligned} 0 = \max_{c,x} \Big[& -\exp(-\rho t - c) - \rho J + J_W\{Wr - c + x[e_0 - (r+a_\theta)s_\theta\theta - (r+a_\eta)\eta]\} + \\ & + \tfrac{1}{2}\sigma_S^2 x^2 J_{WW} + \sigma_{S\theta} x J_{W\theta} + \sigma_{S\eta} x J_{W\eta} \Big] - a_\theta \theta J_\theta - a_\eta \eta J_\eta + \\ & + \tfrac{1}{2}\sigma_\theta^2 J_{\theta\theta} + \tfrac{1}{2}\sigma_\eta^2 J_{\eta\eta} + \sigma_{\theta\eta} J_{\theta\eta}. \end{aligned} \tag{B.2}$$

This gives the optimal consumption and investment policies:

$$c = -\ln J_W = rW + V(D,\mu,\theta,\hat{\mu}) - \ln(r), \tag{B.3}$$

$$x = \{[e_0 - (r+a_\theta)s_\theta\theta - (r+a_\eta)\eta] - \sigma_{S\theta} V_\theta - \sigma_{S\eta} V_\eta\}/(r\sigma_S^2). \tag{B.4}$$

Rewrite the Bellman equation in terms of $V(\theta,\eta)$:

$$0 = r - \rho - r\ln(r) + rV - \frac{1}{2\sigma_S^2}\{[e_0 - (r+a_\theta)s_\theta\theta - (r+a_\eta)\eta] - \sigma_{S\theta} V_\theta - \sigma_{S\eta} V_\eta\}^2 -$$

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[46] It is easy to show that this form of the solution satisfies the transversality condition for $r>0$.

$$+a_\theta V_\theta + a_\eta V_\eta + \frac{1}{2}\sigma_\theta^2(V_\theta^2 - V_{\theta\theta}) + \frac{1}{2}\sigma_\eta^2(V_\eta^2 - V_{\eta\eta}) + \sigma_{\theta\eta}(V_\theta V_\eta - V_{\theta\eta}). \tag{B.5}$$

The form of the Bellman equation in Eq.(B.5) suggests a solution which is quadratic in θ and η:

$$V(\theta,\eta) = \frac{1}{2}v_{\theta\theta}\theta^2 + \frac{1}{2}v_{\eta\eta}\eta^2 + v_{\theta\eta}\theta\eta + v_\theta\theta + v_\eta\eta + v_0. \tag{B.6}$$

This gives a linear demand function for stock:

$$x = f_0 + f_\theta\theta + f_\eta\eta, \tag{B.7}$$

where

$$f_0 = [e_0 - \sigma_{S\theta}v_\theta - \sigma_{S\eta}v_\eta]/(r\sigma_S^2),$$

$$f_\theta = [-(r+a_\theta)s_\theta - \sigma_{S\theta}v_{\theta\theta} - \sigma_{S\eta}v_{\theta\eta}]/(r\sigma_S^2),$$

$$f_\eta = [-(r+a_\eta)s_\eta - \sigma_{S\theta}v_{\theta\eta} - \sigma_{S\eta}v_{\eta\eta}]/(r\sigma_S^2).$$

By substituting Eq.(B.6-7) into Eq.(B.5), the coefficients in $V(\theta,\eta)$ are determined by the following equations:

$$0 = -[-(r+a_\theta)s_\theta - \sigma_{S\theta}v_{\theta\theta} - \sigma_{S\eta}v_{\theta\eta}]^2$$

$$+\sigma_S^2[\sigma_\theta^2 v_{\theta\theta}^2 + 2\sigma_{\theta\eta}v_{\theta\theta}v_{\theta\eta} + \sigma_\eta^2 v_{\theta\eta}^2 + (r+2a_\theta)v_{\theta\theta}]. \tag{B.8.1}$$

$$0 = -[-(r+a_\eta)s_\eta - \sigma_{S\theta}v_{\theta\eta} - \sigma_{S\eta}v_{\eta\eta}]^2$$

$$+\sigma_S^2[\sigma_\theta^2 v_{\theta\eta}^2+2\sigma_{\theta\eta}v_{\theta\eta}v_{\eta\eta}+\sigma_\eta^2 v_{\eta\eta}^2+(r+2a_\eta)v_{\eta\eta}] = 0, \qquad (B.8.2)$$

$$0 = -[-(r+a_\theta)s_\theta-\sigma_{S\theta}v_{\theta\theta}-\sigma_{S\eta}v_{\theta\eta}][-(r+a_\eta)s_\eta-\sigma_{S\theta}v_{\theta\eta}-\sigma_{S\eta}v_{\eta\eta}]+$$

$$+\sigma_S^2[\sigma_\theta^2 v_{\theta\theta}v_{\theta\eta}+\sigma_{\theta\eta}(v_{\theta\theta}v_{\eta\eta}+v_{\theta\eta}^2)+\sigma_\eta^2 v_{\theta\eta}v_{\eta\eta}+(r+a_\theta+a_\eta)v_{\theta\eta}], \qquad (B.8.3)$$

$$0 = -[-(r+a_\theta)s_\theta-\sigma_{S\theta}v_{\theta\theta}-\sigma_{S\eta}v_{\theta\eta}][e_0-\sigma_{S\theta}v_\theta-\sigma_{S\eta}v_\eta]+$$

$$+\sigma_S^2[\sigma_\theta^2 v_{\theta\theta}v_\theta+\sigma_{\theta\eta}(v_{\theta\theta}v_\eta+v_{\theta\eta}v_\theta)+\sigma_\eta^2 v_{\theta\eta}v_\eta+(r+a_\theta)v_\theta], \qquad (B.8.4)$$

$$0 = -[-(r+a_\eta)s_\eta-\sigma_{S\theta}v_{\theta\eta}-\sigma_{S\eta}v_{\eta\eta}][e_0-\sigma_{S\theta}v_\theta-\sigma_{S\eta}v_\eta]+$$

$$+\sigma_S^2[\sigma_\theta^2 v_{\theta\eta}v_\theta+\sigma_{\theta\eta}(v_{\eta\eta}v_\theta+v_{\theta\eta}v_\eta)+\sigma_\eta^2 v_{\eta\eta}v_\eta+(r+a_\eta)v_\eta], \qquad (B.8.5)$$

$$0 = -[e_0-\sigma_{S\theta}v_\theta-\sigma_{S\eta}v_\eta]^2+\sigma_S^2\{\sigma_\theta^2(v_\theta^2-v_{\theta\theta})+2\sigma_{\theta\eta}(v_\theta v_\eta-v_{\theta\eta})+\sigma_\eta^2(v_\eta^2-v_{\eta\eta})+$$

$$2[rv_0+r-\rho-r\ln(r)]\}. \qquad (B.8.6)$$

We can solve Eq.(B.8.1-3) for $v_{\theta\theta}$, $v_{\theta\eta}$ and $v_{\eta\eta}$. This is a set of quadratic equations. The root may not always be unique. We will choose the one that gives the highest value function and the correct limit as σ_D, σ_μ go to zero.

Appendix C: Solution to the Uninformed Investors' Optimization Problem

In this appendix, we solve the optimization problem for an uninformed investor.

As mentioned in Appendix A, the information structure generated by $F^{D,S}(t)$ is equivalent to the information structure generated by the innovation process

$\bar{w}(t)$. The posterior mean and variance are innovated by $\bar{w}(t)$ due to the filtering equation (A.8-10). Since $\bar{w}(t)$ is a Wiener process, the system $[\hat{\theta},\hat{\mu}]$ is Markovian with respect to $F^{\bar{w}}$. Using the $\bar{w}$-representation of the information structure, the the optimization problem of the uninformed investors becomes a Markovian one. Formally, it is similar to that of the informed investors except that filters are used for the unobserved state variables. This result is know as the separation principle.[47]

Let W* be an uninformed investor's wealth, c* his consumption rate, x* his holding of stock and $J^*(W^*;D,\hat{\mu},\hat{\theta};t)$ his value function. By the separation principle, the program is restated as:

$$0 = \underset{c^*,x^*}{\text{Max}} \{ -\exp(-\rho t-c^*) + E[dJ^*(W^*;D,\hat{\mu},\hat{\theta};t)\,|\,I_t^*]/dt \} \qquad (C.1)$$

$$\text{s.t. } dW^* = (rW^*-c^*)dt+x^*[Ddt+dS-rSdt]$$

For an uninformed investor, the expected excess return on the stock conditioned on his information is $[e_0-(r+a_\theta)s_\theta\hat{\theta}]$ since $E[\theta\,|\,I_t^*]=\hat{\theta}$ and $E[\eta\,|\,I_t^*]$ is zero. From the perspective of an uninformed investor, $\hat{\theta}$ is the only state variable that affects the expected excess return on the stock. Hence, his value function only depends on $\hat{\theta}$. Let us assume that an uninformed investor's value function has the form:

$$J^*(W^*;D,\hat{\mu},\hat{\theta};t) = -\exp\{-\rho t-rW^*-V^*(\hat{\theta})\}. \qquad (C.2)$$

---

[47] For a more detailed discussion of the separation principle, see Fleming and Rishel (1975), Whittle (1981), Bensoussan and van Schuppen (1985). Note that the case here is a special case of the well known LQG case in the sense that our objective function does not depend on the unobserved state variables. See also Dothan and Feldman (1986), Detemple (1986), Gennotte (1986).

The corresponding Bellman equation is:

$$0 = \max_{c^*,x^*} \Big[-\exp(-\rho t - c^*) - \rho J + J^*_{W^*}(W^* r - c^* + x^*[e_0 - (r+a_\theta)s_\theta \hat{\theta}]) + \frac{1}{2}\sigma_S^2 x^{*2} J^*_{W^*W^*} + \sigma_S \hat{\sigma}_\theta x^* J^*_{W^*\hat{\theta}} \Big] - a_\theta \hat{\theta} J^*_{\hat{\theta}} + \frac{1}{2}\sigma_{\hat{\theta}}^2 J^*_{\hat{\theta}\hat{\theta}}. \tag{C.3}$$

With the trial form in Eq.(C.2), we have the following first order conditions for c* and x*:

$$c^* = -\ln J^*_{W^*} = rW^* + V^*(\hat{\theta}) - \ln(r), \tag{C.4.1}$$

$$x^* = \{[e_0 - (r+a_\theta)s_\theta \hat{\theta}] - \sigma_S \hat{\sigma}_\theta V^*_{\hat{\theta}}\}/(r\sigma_S^2). \tag{C.4.2}$$

Substituting the first order conditions into the Bellman equation (C.3), we obtain

$$0 = r - \rho - r\ln(r) + rV(\hat{\theta}) + a_\theta \hat{\theta} V^*_{\hat{\theta}} - \{[e_0 - (r+a_\theta)s_\theta \hat{\theta}] - \sigma_S \hat{\sigma}_\theta V^*_{\hat{\theta}}\}^2/(2\sigma_S^2) + \frac{1}{2}\sigma_{\hat{\theta}}^2 (V^{*2}_{\hat{\theta}} - V^*_{\hat{\theta}\hat{\theta}}). \tag{C.5}$$

The above equation has a quadratic solution:

$$V^*(\hat{\theta}) = \frac{1}{2}v^*_{\theta\theta}\hat{\theta}^2 + v^*_\theta \hat{\theta} + v^*_0. \tag{C.6}$$

This leads to a linear stock demand function:

$$x^* = f^*_0 + f^*_\theta \hat{\theta}, \tag{C.7}$$

where $f_0^*=(e_0-\sigma_S\hat{\theta}v_\theta^*)/(r\sigma_S^2)$, $f_\theta^*=[-(r+a_\theta)s_\theta-\sigma_S\hat{\theta}v_{\theta\theta}^*]/(r\sigma_S^2)$. Substituting Eq.(C.6) into the Bellman equation, we obtain the equations for the coefficients in $V^*(\hat{\theta})$, $v_{\theta\theta}^*$, v_θ^*, v_0^*:

$$-[-(r+a_\theta)s_\theta-\sigma_S\hat{\theta}v_{\theta\theta}^*]^2+\sigma_S^2[\sigma_{\hat{\theta}}^2v_{\theta\theta}^{*2}+(r+2a_\theta)v_{\theta\theta}^*] = 0, \tag{C.8.1}$$

$$-[-(r+a_\theta)s_\theta-\sigma_S\hat{\theta}v_{\theta\theta}^*](e_0-\sigma_S\hat{\theta}v_\theta^*)+\sigma_S^2[\sigma_{\hat{\theta}}^2v_{\theta\theta}^*v_\theta^*+(r+a_\theta)v_\theta^*] = 0, \tag{C.8.2}$$

$$-(e_0-\sigma_S\hat{\theta}v_\theta^*)^2+\sigma_S^2[\sigma_{\hat{\theta}}^2(v_\theta^{*2}-v_{\theta\theta}^*)+2[r-r\ln(r)-\rho+rv_0^*] = 0. \tag{C.8.3}$$

Appendix D: Autocorrelation in Excess Returns

In this appendix, we solve Eq.(2.3), (4.7) and (4.8) for Q. We write Eq.(2.3), (4.7) and (4.8) in matrix form

$$dx = edt + Axdt + dw, \tag{D.1}$$

where

$$x = \begin{bmatrix} dQ \\ d\theta \\ d\eta \end{bmatrix}, \quad e = \begin{bmatrix} e_0 \\ 0 \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & -\gamma_\theta s_\theta & -\gamma_\eta s_\eta \\ 0 & -a_\theta & 0 \\ 0 & 0 & -a_\eta \end{bmatrix}, \quad dw = \begin{bmatrix} \sigma_S dw_S \\ \sigma_\theta dw_\theta \\ \sigma_\eta dw_\eta \end{bmatrix}.$$

As shown in Arnold (1974), Eq.(D.1) has the following solution

$$x = e(t-t_0) + x(t_0)\int_{t_0}^{t} \exp\{A(t-s)\}eds + \int_{t_0}^{t} \exp\{A(t-s)\}dw(s). \tag{D.2}$$

Explicitly, θ, η and Q can be expressed as

$$\theta(t) = \theta(t_0)\exp\{-a_\theta(t-t_0)\} + \int_{t_0}^{t} \exp\{-a_\theta(t-s)\}\sigma_\theta dw_\theta(s), \tag{D.3}$$

$$\eta(t) = \eta(t_0)\exp\{-a_\eta(t-t_0)\} + \int_{t_0}^{t} \exp\{-a_\eta(t-s)\}\sigma_\eta dw_\eta(s), \tag{D.4}$$

$$Q(t) = Q(t_0) + e_0(t-t_0) + \gamma_\theta s_\theta[\theta(t)-\theta(t_0)] + \gamma_\eta s_\eta[\eta(t)-\eta(t_0)]$$

$$+ \int_{t_0}^{t} [\sigma_S dw_S - \gamma_\theta s_\theta \sigma_\theta dw_\theta - \gamma_\eta s_\eta \sigma_\eta dw_\eta]. \tag{D.5}$$

Define $G(\tau,a)=1-\exp(-a\tau)$. Then

$$\mathrm{Var}_t[Q(t+\tau)-Q(t)] = \sigma_S^2 \int_t^{t+\tau} ds + 2(\gamma_\theta s_\theta)(\gamma_\eta s_\eta)\sigma_{\theta\eta} \int_t^{t+\tau} G(t+\tau-s,a_\theta)G(t+\tau-s,a_\eta)ds$$

$$+ (\gamma_\theta s_\theta)^2\sigma_\theta^2 \int_t^{t+\tau} [G(t+\tau-s,a_\theta)]^2 ds + (\gamma_\eta s_\eta)^2\sigma_\eta^2 \int_t^{t+\tau} [G(t+\tau-s,a_\eta)]^2 ds$$

$$- 2(\gamma_\theta s_\theta)\sigma_{S\theta} \int_t^{t+\tau} G(t+\tau-s,a_\theta)ds - 2(\gamma_\eta s_\eta)\sigma_{S\eta} \int_t^{t+\tau} G(t+\tau-s,a_\eta)ds$$

Define

$$q_0 = \sigma_S^2+(\gamma_\theta s_\theta)^2\sigma_\theta^2+(\gamma_\eta s_\eta)^2\sigma_\eta^2-2\gamma_\theta s_\theta\sigma_{S\theta}-2\gamma_\eta s_\eta\sigma_{S\eta}+2\gamma_\theta\gamma_\eta s_\theta s_\eta\sigma_{\theta\eta},$$

$$q_1 = (2/a_\theta)[(\gamma_\theta s_\theta)^2\sigma_\theta^2-\gamma_\theta s_\theta\sigma_{S\theta}+\gamma_\theta\gamma_\eta s_\theta s_\eta\sigma_{\theta\eta}],$$

$$q_2 = (2/a_\eta)[(\gamma_\eta s_\eta)^2\sigma_\eta^2-\gamma_\eta s_\eta\sigma_{S\eta}+\gamma_\theta\gamma_\eta s_\theta s_\eta\sigma_{\theta\eta}],$$

$$q_3 = (\gamma_\theta s_\theta)^2 \mathrm{Var}(\theta),\ q_4 = (\gamma_\eta s_\eta)^2 \mathrm{Var}(\eta),\ q_5 = 2\gamma_\theta \gamma_\eta s_\theta s_\eta \mathrm{Cov}(\theta,\eta).$$

Here, $\mathrm{Var}(\theta)=\sigma_\theta^2/(2a_\theta)$, $\mathrm{Var}(\eta)=\sigma_\eta^2/(2a_\eta)$, and $\mathrm{Cov}(\theta,\eta)=\sigma_{\theta\eta}/(a_\theta+a_\eta)$. Then,

$$\mathrm{Var}_t[Q(t+\tau)-Q(t)] = q_0\tau - q_1 G(\tau,a_\theta) - q_2 G(\tau,a_\eta) +$$

$$+ q_3 G(\tau,2a_\theta) + q_4 G(\tau,2a_\eta) + q_5 G(\tau,a_\theta+a_\eta).$$

Given Eq.(D.5), we can also calculate the covariance between the excess returns for two neighboring holding periods, $\mathrm{Cov}[Q(t+\tau)-Q(t),\ Q(t)-Q(t-\tau)]$. Define

$$\Delta_\theta = \gamma_\theta s_\theta[(\sigma_{S\theta}/a_\theta)-(\gamma_\theta s_\theta)\mathrm{Var}(\theta)-(\gamma_\eta s_\eta)(a_\eta/a_\theta)\mathrm{Cov}(\theta,\eta)],$$

$$\Delta_\eta = \gamma_\eta s_\eta[(\sigma_{S\eta}/a_\eta)-(\gamma_\eta s_\eta)\mathrm{Var}(\eta)-(\gamma_\theta s_\theta)(a_\theta/a_\eta)\mathrm{Cov}(\theta,\eta)].$$

The covariance can be expressed as follows:

$$\mathrm{Cov}[Q(t+\tau)-Q(t),\ Q(t)-Q(t-\tau)] = -\Delta_\theta[G(\tau,a_\theta)]^2 - \Delta_\eta[G(\tau,a_\eta)]^2.$$

Furthermore, it can be shown that

$$\mathrm{Cov}[\eta(t+\tau)-\eta(t),\ Q(t)-Q(t-\tau)] = -\Delta_\eta[G(\tau,a_\eta)]^2.$$

Since $E[\eta(t+\tau)-\eta(t)\,|\,I_t^*]=0$, $E[\eta(t+\tau)-\eta(t)\,|\,Q(t)-Q(t-\tau)] = 0$. Thus,

$$\mathrm{Cov}[\eta(t+\tau)-\eta(t),\ Q(t)-Q(t-\tau)] = 0.$$

Therefore,

$$\mathrm{Cov}[Q(t+\tau)-Q(t),\ Q(t)-Q(t-\tau)] = -\Delta_\theta[G(\tau,a_\theta)]^2$$

Appendix E: Unconditional Variance of Stock Prices

The price of the stock satisfies the stochastic differential equation Eq.(4.7). Define $m_1=a_D s_{D0}/(a_\mu-a_D k)$, $m_2=s_{\mu 0}-m_1$. Eq.(4.7) has the following solution:

$$\begin{aligned} S(t) = {} & S(t_0) + s_{D0}D(t_0)\exp(-a_D\tau) + m_2\mu(t_0)\exp(-a_\mu\tau) \\ & + s_\theta\theta(t_0)\exp(-a_\theta\tau) + s_\eta\eta(t_0)\exp(-a_\eta\tau) \\ & + \int_{t_0}^{t} \{ s_{D0}\exp[-a_D(t-s)]\sigma_D dw_D + (m_1\exp[-a_D(t-s)]+m_2\exp[-a_\mu(t-s)])\sigma_\mu dw_\mu \\ & \quad + s_\theta\exp[-a_\theta(t-s)]\sigma_\theta dw_\theta + s_\eta\exp[-a_\eta(t-s)]\sigma_\eta dw_\eta \}. \end{aligned} \tag{E.1}$$

From Eq.(E.1), we can calculate the unconditional variance of S:

$$\begin{aligned} \mathrm{Var}(S) = {} & (s_{D0}\sigma_D)^2/(2a_D) + [m_1^2/(2a_D)+m_2^2/(2a_\mu)+2m_1m_2/(a_D+a_\mu)]\sigma_\mu^2 \\ & + (s_\theta\sigma_\theta)^2/(2a_\theta) + (s_\eta\sigma_\eta)^2/(2a_\eta) + 2s_{D0}s_\eta\sigma_{D\eta}/(a_D+a_\eta) \\ & + 2[m_1/(a_D+a_\eta)+m_2/(a_\mu+a_\eta)]s_\eta\sigma_{\mu\eta} + 2s_\theta s_\eta\sigma_{\theta\eta}/(a_\theta+a_\eta). \end{aligned}$$

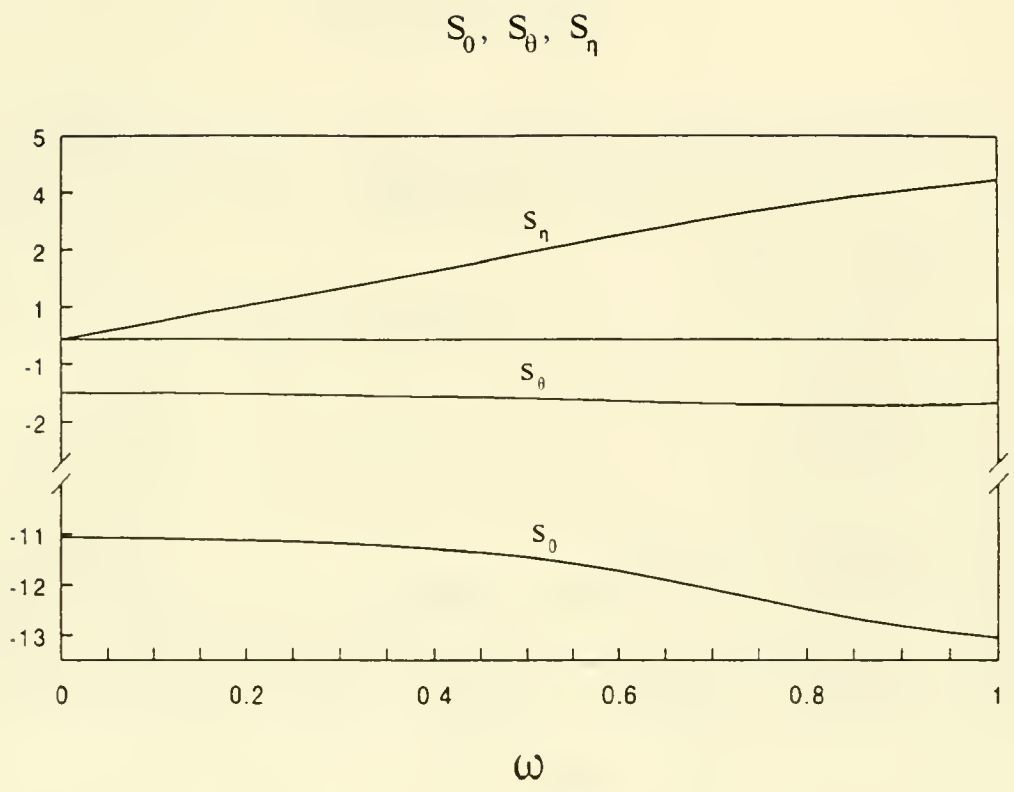

FIGURE 1 s_0, s_θ, s_η plotted against ω

$r = 0.05$, $\rho = 0.2$, $b = 0.85$, $k = 1.2$,
$a_D = 0.1$, $a_\mu = 0.1$, $a_\theta = 0.4$,
$\sigma_D = 0.4$, $\sigma_\mu = 0.6$, $\sigma_\theta = 1.0$

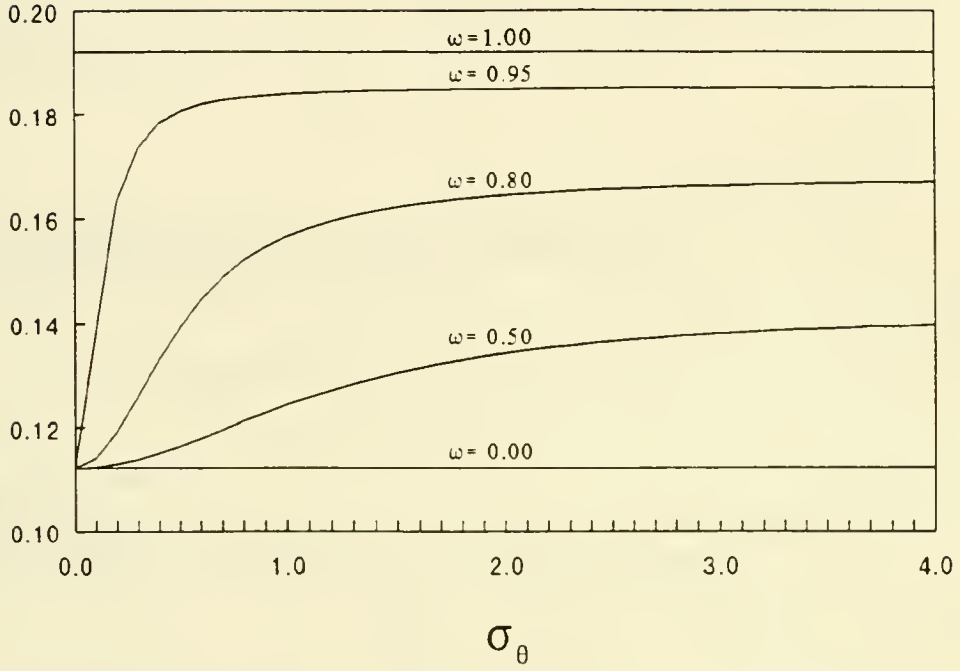

FIGURE 2 $\frac{E[dQ]}{dt} = -rs_0$, plotted against σ_θ

$r = 0.8$, $\rho = 2.0$, $b = 0.5$, $k = 1.0$,
$a_D = 0.5$, $a_\mu = 0.1$, $a_\theta = 0.5$,
$\sigma_D = 0.4$, $\sigma_\mu = 0.5$, $\sigma_\theta = 1.0$

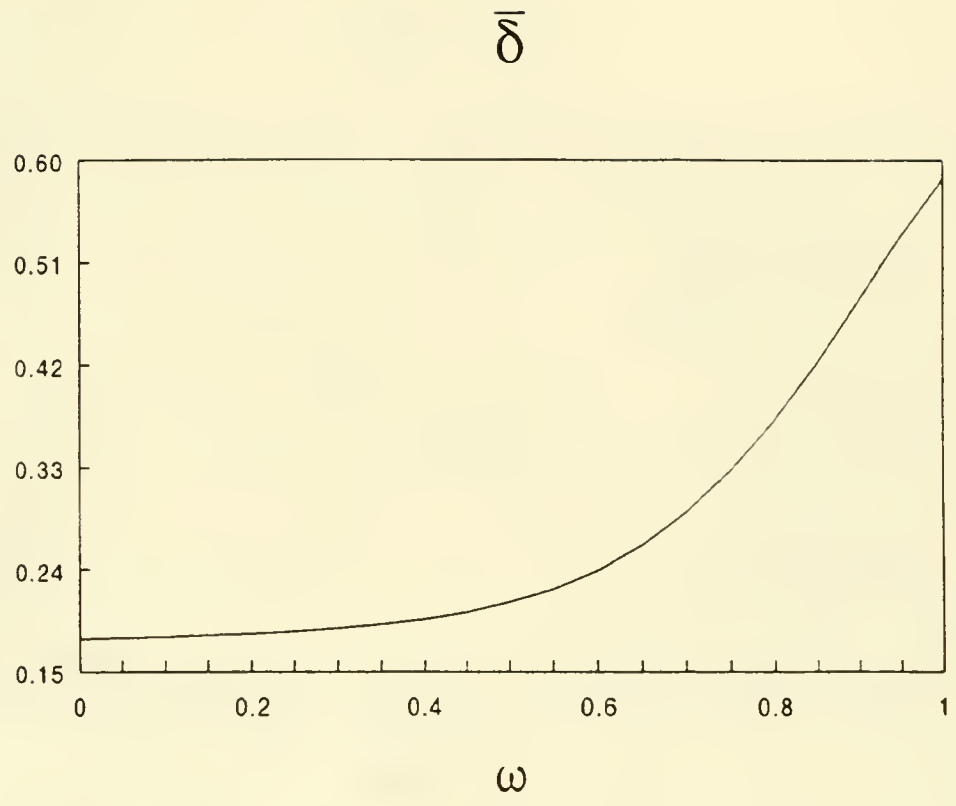

FIGURE 3 $\overline{\delta}$ plotted against ω

$r = 0.05$, $\rho = 0.2$, $b = 0.85$, $k = 1.2$,
$a_D = 0.1$, $a_\mu = 0.1$, $a_\theta = 0.4$,
$\sigma_D = 0.4$, $\sigma_\mu = 0.6$, $\sigma_\theta = 1.0$

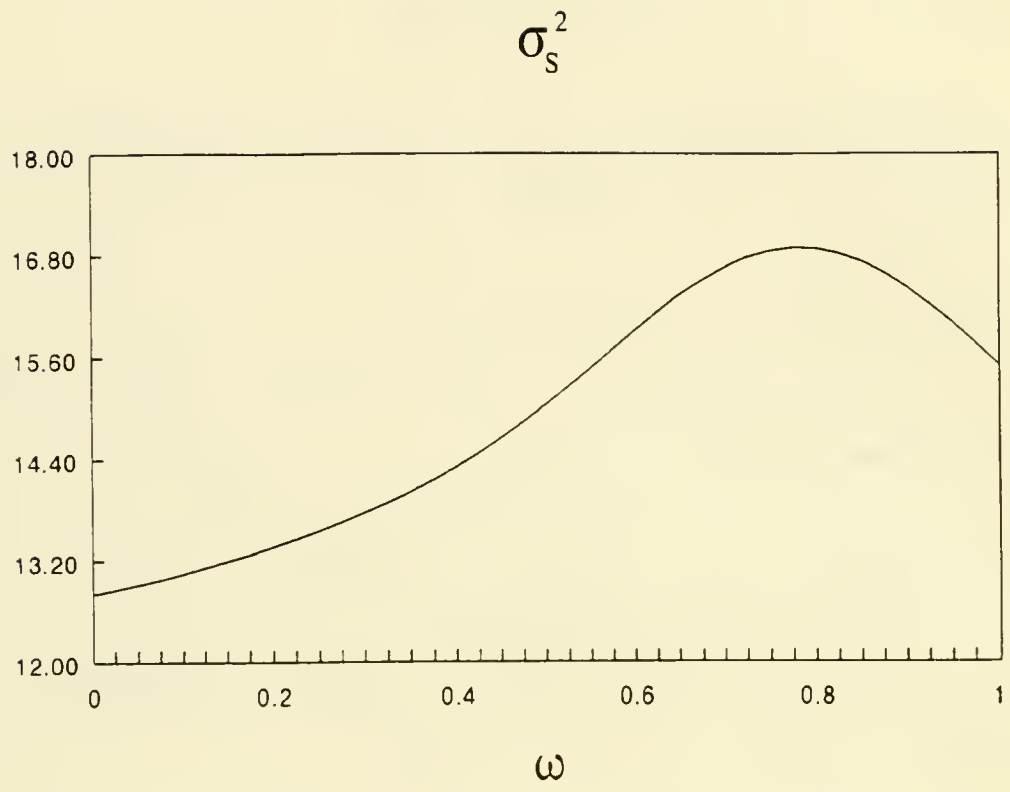

FIGURE 4 σ_S^2 plotted against ω

$r = 0.05$, $\rho = 0.2$, $b = 0.85$, $k = 1.2$,
$a_D = 0.1$, $a_\mu = 0.1$, $a_\theta = 0.4$,
$\sigma_D = 0.4$, $\sigma_\mu = 0.6$, $\sigma_\theta = 1.0$

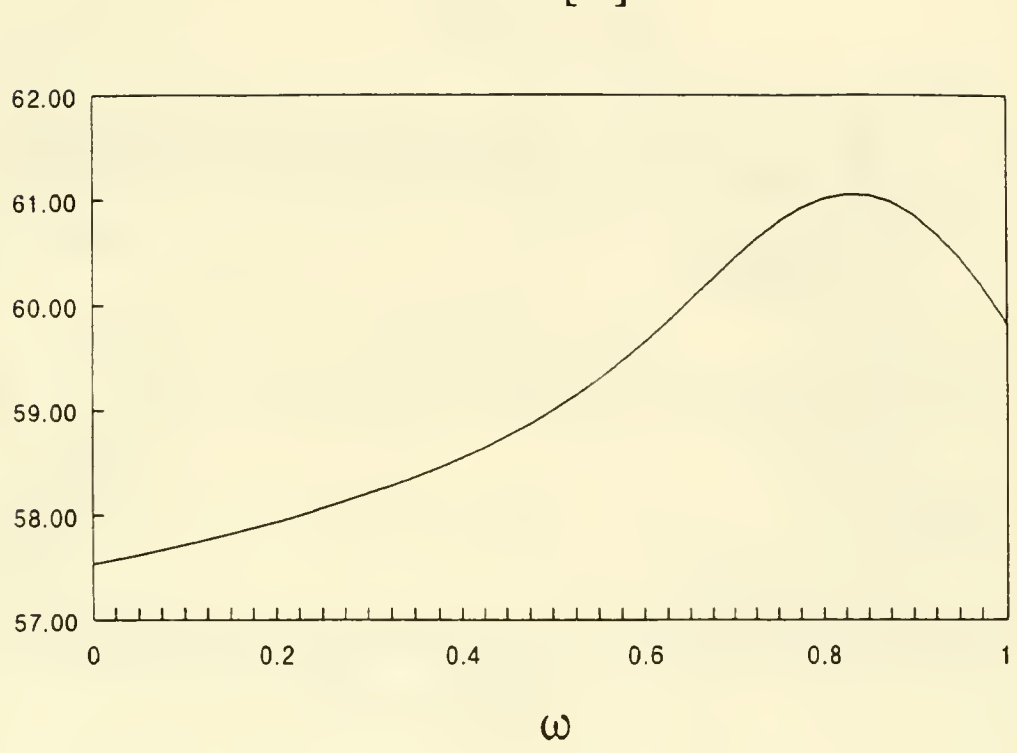

FIGURE 5 *Var[S]* plotted against ω

$r = 0.05$, $\rho = 0.2$, $b = 0.85$, $k = 1.2$,
$a_D = 0.1$, $a_\mu = 0.1$, $a_\theta = 0.4$,
$\sigma_D = 0.4$, $\sigma_\mu = 0.6$, $\sigma_\theta = 1.0$

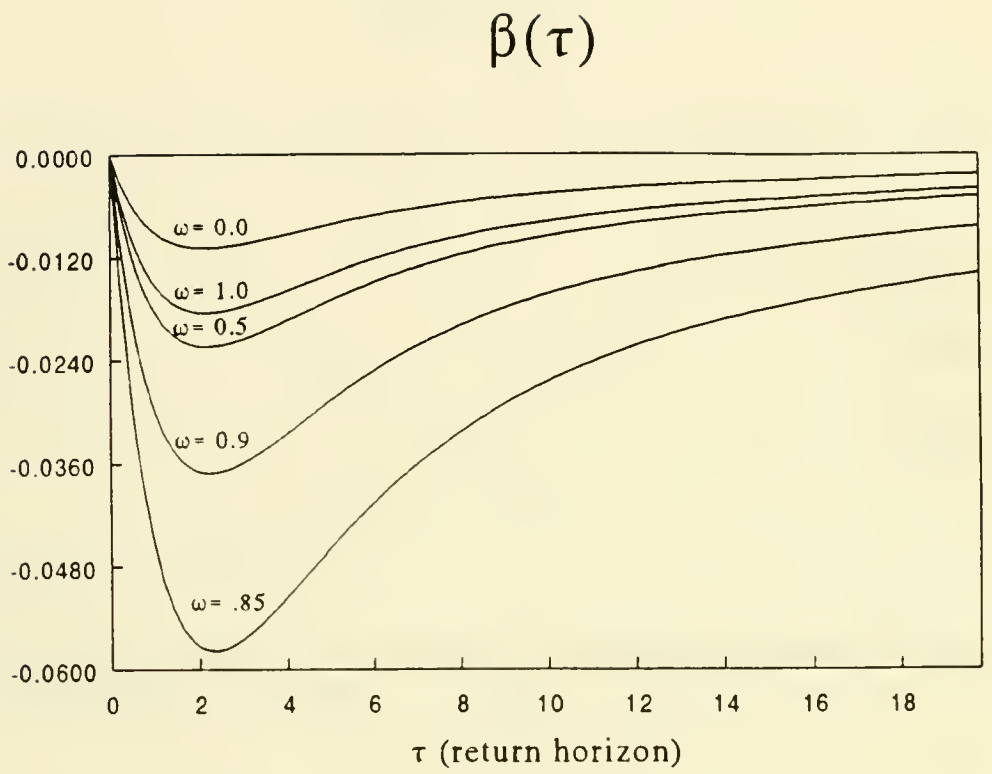

FIGURE 6 $\beta(\tau)$ plotted against τ

$r = 0.05$, $\rho = 0.4$, $b = 0.5$, $k = 0.8$,
$a_D = 0.1$, $a_\mu = 0.01$, $a_\theta = 0.6$,
$\sigma_D = 1.0$, $\sigma_\mu = 1.0$, $\sigma_\theta = 0.2$

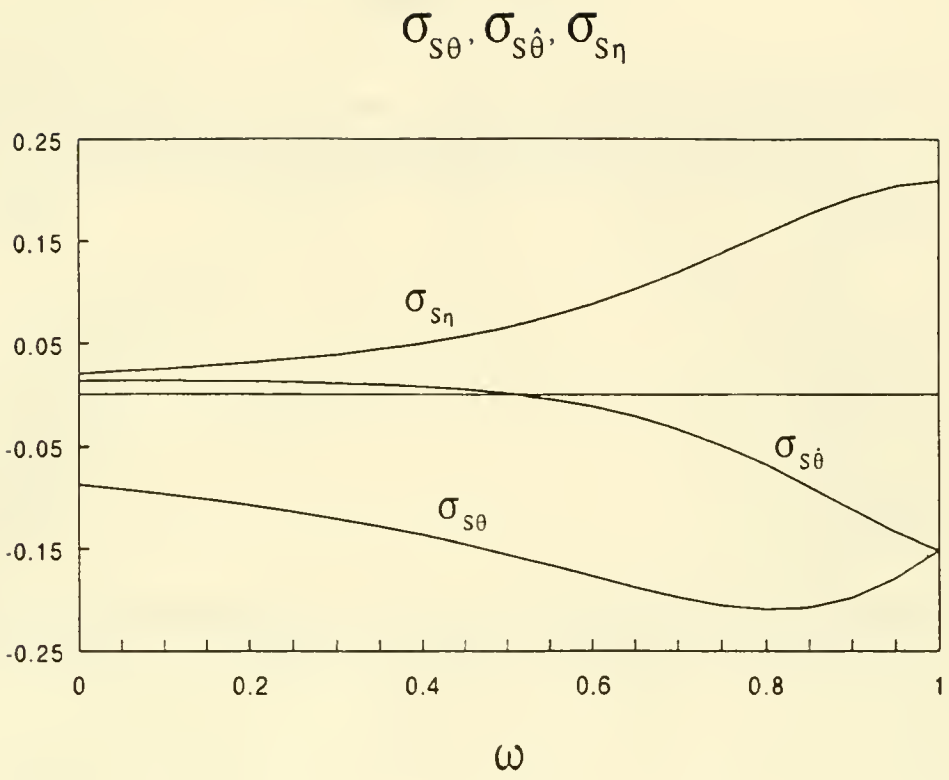

FIGURE 7.1 $\sigma_{S\theta}$, $\sigma_{S\hat{\theta}}$, $\sigma_{S\eta}$ plotted against ω

$r = 0.8$, $\rho = 2.0$, $b = 0.5$, $k = 1.0$,
$a_D = 0.5$, $a_\mu = 0.1$, $a_\theta = 0.5$,
$\sigma_D = 0.4$, $\sigma_\mu = 0.5$, $\sigma_\theta = 1.0$

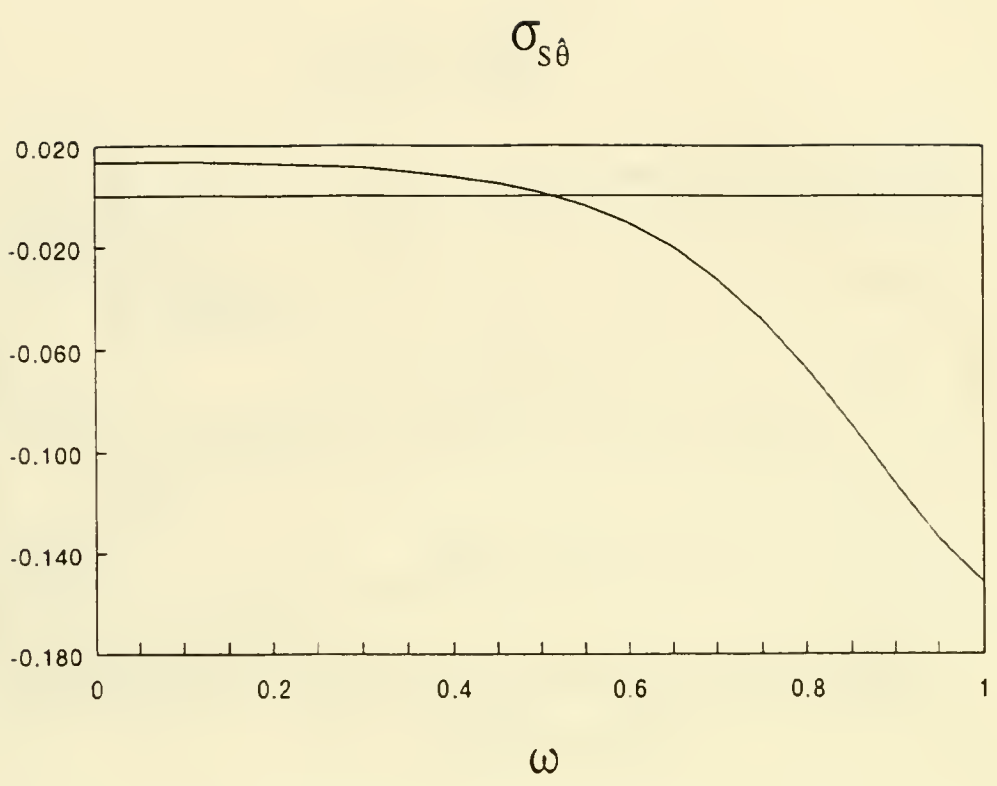

FIGURE 7.2 $\sigma_{s\theta}$ plotted against ω

$r = 0.8$, $\rho = 2.0$, $b = 0.5$, $k = 1.0$,
$a_D = 0.5$, $a_\mu = 0.1$, $a_\theta = 0.5$,
$\sigma_D = 0.4$, $\sigma_\mu = 0.5$, $\sigma_\theta = 1.0$

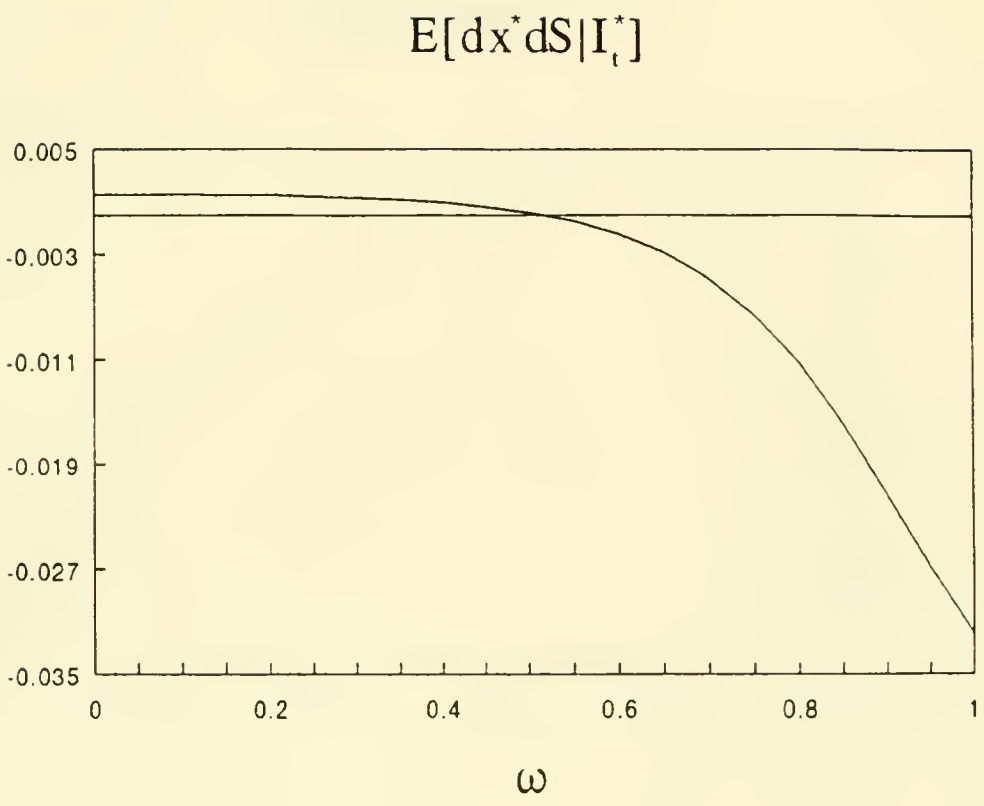

FIGURE 8.1 $E[dx^{*}dS|I_t^{*}]$ plotted against ω

$r = 0.8$, $\rho = 2.0$, $b = 0.5$, $k = 1.0$,
$a_D = 0.5$, $a_\mu = 0.1$, $a_\theta = 0.5$,
$\sigma_D = 0.4$, $\sigma_\mu = 0.5$, $\sigma_\theta = 1.0$

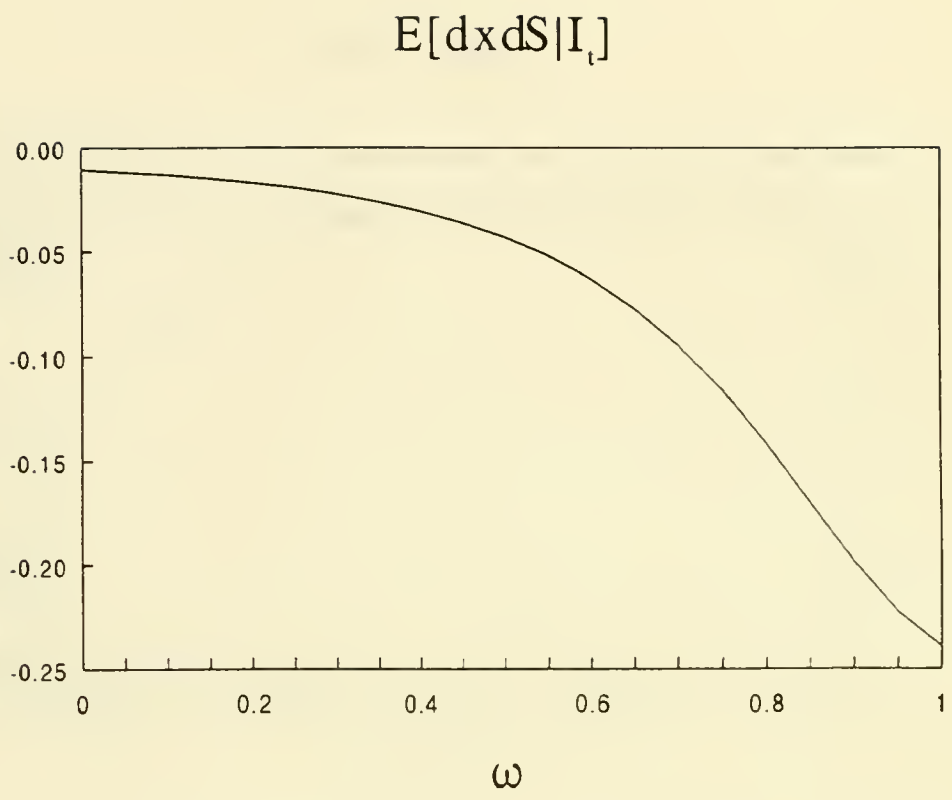

FIGURE 8.2 E[*dxdS*|I_t] plotted against ω

r = 0.8, ρ = 2.0, b = 0.5, k = 1.0,
a_D = 0.5, a_μ = 0.1, a_θ = 0.5,
σ_D = 0.4, σ_μ = 0.5, σ_θ = 1.0

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